Y9.GEH2.1: Cost to benefit analysis of the FREEDM system: tradeoffs in designs and evaluation of alternative designs

Project Leader: G. Heydt

Students: A. Dinakar, Ms. Apurva Raman

1. Project Goals

The goal of this project is to evaluate alternative designs for the FREEDM distribution system. The cost and benefit of components are to be evaluated. Uncertainty in the cost and benefit data are modelled in a stochastic way. The broader impact of this work is a probabilistic cost to benefit evaluation for engineering projects in general.

2. Role in Support of Strategic Plan

The project is in support of the cost to benefit analysis task.

3. Fundamental Research, Technological Barriers and Methodologies

Previously unreported work from Y8

A masters thesis was prepared showing alternatives for the basic configuration of the FREEDM distribution system. This focuses on the number of distribution primaries to serve a given residential load. The lower load currents involved in several parallel distribution primaries are favored to decrease the cost of interruption devices. An evaluation of the reduction of the number of fault interruption devices versus reduction in circuit performance was completed.

In [1], an assessment is given of the role of residential loads versus non-residential loads in the general study of peak reduction, energy use, and energy management. Although the focus is not on the FREEDM system, the results appear to favor the need for more emphasis on industrial and commercial electric load management.

New work reported in Y9

The concept proposed in this FREEDM project is to incorporate probabilistic modeling in the cost to benefit analysis of the FREEDM system. The central concept is to model all of these uncertain parameters as random variables of assumed probability density functions. The deterministic cost to benefit analysis is then a subset of the probabilistic formulation with the various uncertain parameters modeled as simply parameters with impulse probability density functions. These impulses are located at their expected (estimated) values. With that formulation, the problem is purely deterministic, and it is not possible to calculate such indicators as the probability of achieving a favorable cost to benefit ratio within five years (for example), or the conditional expectation of payback period given that the value of peak demand power will be at a given level. In this part of the FREEDM cost / benefit research project, the probabilistic approach is proposed, formulated, and applied. The broader impact of the work is that the probabilistic cost / benefit analysis is applicable generally to any engineering problem in which costs and benefits can be monetized in a probabilistic way. In this section, a short summary is given for the probabilistic approach to the cost/benefit problem. Two reports [2,3] are a more formal presentation of the work. Both references are products of the FREEDM center.

Clearly an *Achilles' heel* of the probabilistic cost / benefit analysis is the availability of statistically accurate data on the costs of equipment, electric energy, capital, inflation, and other uncertain data; and also the estimation of the benefits of a given design (e.g., monetization of reduction in active power losses, improvement in reliability, reduction of system average duration and frequency indices, increased revenue attained). There are tools to estimate these uncertainties, however. Some of these tools are: nationally available statistical repositories, expert estimates, manufacturers' contemporary data, large scale studies and economic model estimates.

In order to formulate the cost to benefit analysis as a probabilistic problem, it is necessary to estimate the *probability density functions* of the 'input variables' (i.e., the uncertain parameters in the analysis). An appeal to the central limit theorem might suggest the use of gaussian density functions, and in such a case, the mean and variance of the Gaussian density determine the probability density function. In a cost / benefit analysis in which equipment costs and operational benefits are estimated as ensembles of data of considerable size, the modeling of the input parameter probability density functions may be approached with greater sophistication and accuracy. The probabilistic cost / benefit formulation is conveniently set up as a problem of the evaluation of the probability density of the multivariate vector *X* which is a vector valued function *H* of the multivariate vector *Y*,

$$X = H(Y). \qquad (1)$$

Let the dimension of *X* and *Y* be the same, although this requirement is easily removed through the use of auxiliary variables. The jacobian matrix of (1) is,

$$J = \begin{bmatrix} \frac{\partial X_1}{\partial Y_1} & \cdots & \frac{\partial X_1}{\partial Y_n} \\ & \cdots & \\ \frac{\partial X_n}{\partial Y_1} & \cdots & \frac{\partial X_n}{\partial Y_n} \end{bmatrix} \qquad (2)$$

where *n* is the common dimension of vectors *X* and *Y*. Assume that the joint probability densities of the Y_i are known (and denoted as $f_{yiyj}(y_i, y_j)$). Then the well known formula to obtain the probability density of vector *X* (namely $f_{xixj}(x_i, x_j)$) is,

$$f_{xixj}(x_i, x_j) = \sum_q \frac{f_{yiyj}(yq)}{det(J)_{yq}} \qquad (3)$$

where the indicated sum is taken over all solutions *q* of (1) (if there is only one solution for *Y* in (1), then *q* is simply that one solution); and *det*(.) refers to the determinant. In (3), *yq* means the elements of vector *Y* evaluated at the solution *q*.

The foregoing discussion gives the mathematical structure of finding the probability density function of multivariates which are themselves functions of other multivariates. In (1), the vector valued function *H* may be nonlinear and the calculation of the inverse function, e.g., $Y = H'(X)$, may be difficult to obtain. It may be necessary to resort to numerical methods to obtain the inverse function *H'*. Further, the jacobian matrix in (2) may contain complicated functions and expressions, and these lead to integrals in (3) that are difficult to evaluate literally. Even the straightforward calculation of the payback time *Y* associated with a cost *C* and benefit *B* expressed as a ratio,

$$Y = C/B \qquad (4)$$

results in a rather complex formulation – even when the probability densities of *C* and *B* are simple forms such as normally distributed. In (4), *C* is the total investment cost of the project, and *B* is the benefits accrued per year. In the stochastic case, (4) is sometimes termed a 'ratio distribution'. In the uncorrelated normally distributed case, the probability density of *Y* becomes a complicated function of arctangents. A solution to the cited complexity is to simply evaluate (4) numerically at sample values: this is the Monte Carlo method. The advantage of the Monte Carlo approach is that the nonlinear functions and the indicated integrals give far fewer computational problems. The probabilistic formulation shown above gives an estimate of the probability density of such parameters as payback period, profit, and expense.

The concept of a stochastic cost / benefit analysis is illustrated by two examples denominated as Examples A and B. The first example utilizes the system theoretic approach for calculation, i.e., (1) – (4). Because the indicated calculations involve complicated integrals, relatively simple probability density functions are illustrated, namely normally distributed cost and benefit. The second example (B) uses the computationally efficient Monte Carlo method with probabilistic data taken from an actual proposed application in distribution engineering. The latter is a future distribution system that is based on electronic components. In Example A, the simple model (4) is used with normally distributed values of both cost *C* and benefit *B*. There is some logic in the examination of the normally distributed case: because both cost and benefit consist of a sum of a number of disparate elements, it is reasonable to appeal to a weak form of the *central limit theorem*. The application of this theorem suggests that both *C* and *B* are approximately normal. Unfortunately, in many applications, these variates are not statistically independent. The use of normal statistics has the convenience that the statistical mean and standard deviation of the variates fully determines their probability density function. For Example A, Table I shows the statistics of *C* and *B*. This is a case in which $\mu_B \gg \sigma_B$ and the benefit is effectively positive over its entire range. The correct expression for the probability density of this ratio distribution is,

$$f_Y(Y) = \frac{1}{\sqrt{2\pi}} \frac{\mu_B\sigma_C^2 + \mu_C\sigma_B^2 Y}{\sqrt{\sigma_C^2 + \sigma_B^2 Y^2}} \exp(\frac{-0.5(\mu_C - \mu_B Y)^2}{\sigma_C^2 + \sigma_B^2 Y^2}) \quad (5)$$

The application of the literal form (10) gives the results shown in Fig. 1. In this case, the mean and standard deviation of the payback period are numerically calculated as $\mu_Y = 5.03\ years$, $\sigma_Y = 0.82\ year$. A simple deterministic calculation of the payback period, i.e., working with only expected values $Y = \mu_C/\mu_B$ gives 4.73 years but the standard deviation of *Y* is unavailable. Using (5) it is possible to evaluate the confidence in the calculation of the expected payback period versus the confidence in the 'input data' (i.e., standard deviation of *C* and *B*).

Table I Cost and benefit statistics: Example A

	Cost *C*	Benefit *B*
Mean, *μ*	26000 $	5500 $/y
Standard deviation, *σ*	1250 $	750 $/y

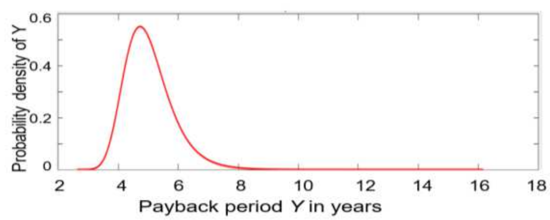

Fig. 1 Probability density function of the payback period *Y* for the Example A. This is found using a system theoretic approach, and formula (5).

A Monte Carlo example is shown here as Example B. Fig. 2 shows a doubly fed FREEDM distribution feeder. The configuration is a 'next generation' FREEDM design in which solid state switching components are used to facilitate control, protection and the integration of renewable resources (e.g., via a solid state transformer with a DC port). Fig. 2 indicates a large number of fault interruption devices (FIDs) but considered in this design are fewer FIDs and the number of FIDs are 'traded off' with reliability. That is, the cost of high speed interruption devices are traded off with the monetization of the enhancement in reliability. Envisioned here is a 15 kV class, three phase design with a large number of 8660 V single phase laterals. Conventional circuit breakers protect the feeder at each end. This example distribution circuit has a total connected load of 10 MVA, and each three phase feeder is rated 3.33 MVA. This design would accommodate 400 residential services averaging 25 kVA each. For example purposes, the probability density $f_C(C)$ is assumed to be uniform, and this is indicated in Table II with representative values of mean and standard deviation of *C*. The statistics of *C* depend on the number of FIDs used (e.g., two single phase FIDs at each single phase lateral root as shown in Fig. 2, or perhaps fewer FIDs placed at alternate roots of the laterals). By this approach, one can assess the system response versus the investment in components. This is a critical issue in the FREEDM system. In Example B, if only the mean values *E*(*C*) and *E*(*B*) were used to find the payback period, *E*(*C*)/*E*(*B*) is 15.385 years (as compared to 15.623 years in a Monte Carlo simulation of the same example.) In Example B, if the cost estimates could be refined, and the standard deviation of *C* reduced by 10%, it is found that the mean payback period is 15.599 years and the standard deviation of *Y* is reduced by 7.4% to 3.719 years. Various conditional expectations and other statistics may be readily found numerically.

4. Achievements

A main project achievement of this study is that it is possible to obtain additional information in a cost to benefit analysis through the use of probabilistic modeling. The advantages of the probabilistic approach include improved repeatability of the calculations, availability of expected values of key cost / benefit parameters, and an estimation of the confidence in the calculation. Instead of ignoring uncertainty in a cost to benefit analysis, the uncertainty is included as best as possible. A disadvantage of the approach is the difficulty in obtaining statistically accurate 'input data'. The probabilistic approach has been illustrated for the case of a next generation power distribution system based on semiconductor control.

The achievement in the FREEDM project is that the indicated stochastic cost to benefit analysis has been applied to the FREEDM distribution system. The details are reported in [2, 3].

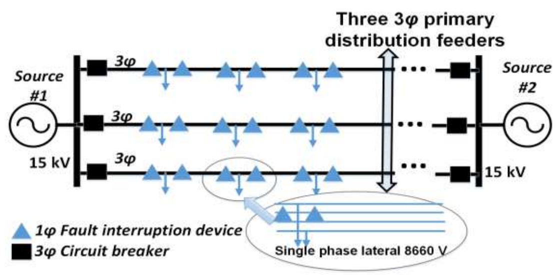

Fig. 2 A next generation FREEDM distribution feeder configuration with fault interruption devices inserted to enhance reliability. Three primary feeders are doubly fed.

Table II Estimated probabilistic models for costs and benefits for Example B

Monetized parameter	Probabilistic model	Mean value	Standard deviation
Cost, *C*	Uniform density	2.000 M$	440 k$
Benefits per year, *B*	Gaussian density	130.0 k$/y	16.0 k$/y

5. Other Relevant Work Being Conducted Within and Outside of the ERC

In the preparation of a report on this work in [2], IEEE organizers indicate a high level of interest in the subject of stochastic models in cost / benefit analysis. It appears that this is a new approach that has not been used or researched elsewhere.

6. Milestones and Deliverables

Q3 (9/30/2016) – Report on the development of accurate probabilistic models for costs - Done

Q4 (12/31/2016) – Report on the use of statistical methods and historical data in connection with the FREEDM benefits identified in Y8, to obtain statistical models of FREEDM benefits - Done

Q1 (3/31/2017) – Completion of tasks on integrating results into the cost / benefit analysis and reporting at NSF site visit 2017 – in progress

Q2 (6/30/2017) – A final report in the form of a MSEE thesis on all results of the analysis – Drafted.

7. Plans for Next Five Years

Plans include the completion of this present work as follows:

- Use system theoretic studies based on functions of several variables to evaluate cost / benefit indices, e.g., apply well known techniques of the calculation of the probability density of functions of several stochastic variables. This application will be used to obtain the probabilistic models of cost / benefit indices.
- Integration with cost / benefit analysis, e.g., to use the results of the cited other tasks in the existing cost / benefit analysis already drafted. And to use optimization techniques to obtain the best cost / benefit under given constraints of specified reliability.

8. Member Company Benefits

- The cost to benefit analysis method appears to be a new approach, and this would give persons who are responsible for cost / benefit studies an alternative, realistic calculation method.
- The methods studied give a realistic analysis of the costs and benefits of the FREEDM system, including uncertainty in the dollar values assumed in the study.

9. References

[1] G. Heydt, "Implementation of smart grid objectives among distribution system residential, commercial, and industrial loads," *Proc. North American Power Symposium*, Denver CO, October, 2016.

[2] G. Heydt, A. Dinakar, "A probabilistic formulation for cost / benefit analysis in power distribution engineering," *Proc. IEEE Power and Energy Society, General Meeting,* Chicago, June, 2017.

[3] A. Dinakar, "A probabilistic cost to benefit assessment of a next generation electric power distribution system," Masters Thesis, Arizona State University, December, 2016.