The Probabilistic Evaluation of Net Present Value of Electric Power Distribution Systems Based on the Kaldor-Hicks Compensation Principle

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***Abstract*—The decision to proceed with a distribution engineering expansion project is often preceded by some form of cost to benefit analysis. The Kaldor-Hicks compensation principle is a criterion that assists in this analysis by the evaluation of *net present value* over the expected project life. This paper discusses this principle applied to power distribution systems. A probabilistic formulation is proposed to capture uncertainty in cost and benefit data. In effect, the approach models ranges of value of project parameters. While the method does not determine whether to proceed with a given project, it does give a measure of the value of the engineering economic efficiency. The method is especially valuable for cases of 'next generation' systems, and this is illustrated in the paper.**

***Index Terms*—Cost benefit analysis; distribution expansion; Kaldor-Hicks compensation principle; net present value; payback period; engineering economics; power distribution engineering.**

I. Net Present Value and an Introduction to its Probabilistic Formulation

THE NET PRESENT VALUE of assets of a power engineering project, in one formulation, may be defined as *NPV*,

$$NPV = B_o - C_o + \sum_{k=1}^{N} \frac{B_k - C_k}{(1+\delta_k)^k} \qquad (1)$$

where the B_k are dollar benefits accrued in interval k of the project, C_k are the costs expended in that interval, δ_k is the discount factor in interval k, and there are $N + 1$ intervals in the project. The B_o and C_o terms are initial returns and investments. The time intervals in the project are ΔT in length, and therefore the project duration is assumed to be $(N+1)\Delta T$. The *NPV* is widely used in diverse applications as a measure of the economic feasibility of a project. Kaldor and Hicks in early papers on the formulation of economic policy [1, 2] led ultimately to a simple criterion, $NPV > 0$, to assess whether a project is economically feasible over the time span $[0, N]$. While this criterion does not capture many practical considerations, it does give some insight into the economic efficiency and validity of an investment. The basic Kaldor-Hicks principle has been applied in a wide range of applications such as the investment of infrastructure for earthquake damage mitigation [3], maintenance scheduling [4], load / process scheduling in operations [5], and public infrastructure / environmental improvements [6].

Stæhr gives a useful tutorial for applications in [7] and this tutorial outlines the way that the original Kaldor and Hicks papers led to the compensation principle. In Stæhr's tutorial, the author notes that the NPV index does not capture all the relevant information on the feasibility and public acceptability of an infrastructure project. In this sense, like many other engineering indices, one must use caution not to assign excessive weight to NPV in the decision making processes. One commonality in the application areas of the Kaldor-Hicks compensation principle is the use of cost – benefit analysis to formulate policies that 'harden' and improve civil infrastructures (for example, adoption of designs and policies to accommodate failures, disasters [8], and economic losses due to nominal operation). Power distribution system expansion and improvements fall into a similar category as these cited infrastructural projects. For example the tradeoffs between overhead and underground distribution engineering policies [9] are used to either enhance existing infrastructures or design new infrastructures. The role of probabilistic cost / benefit analysis in the competitive environment in the electric power industry is documented in [10].

There are alternative ways to assess cost / benefit tradeoffs and the literature contains many application areas, e.g., [11, 12]. Most (if not all) of these applications focus on civil infrastructures, and connections with public funding, taxes, reliability, and economic efficiency (e.g., the Brownfield redevelopment concept which integrates several of these technologies in an 'enhancement of public good' concept [13]). The role of a probabilistic formulation of cost / benefit analysis has migrated into main stream engineering economic studies as evidenced by [14]. One commonly used index of economic efficiency is the 'payback period', *Y*. This index is used to determine how long one must wait until an economic investment begins to yield a financial return. It its simplest form,

$$Y = \frac{\sum C_k}{\sum B_k} \qquad (2)$$

where the total project costs are in the numerator (summed over all values of time interval, k), and the denominator is the total annual benefits. With B_k in dollars per year, Y will be the payback period in years. Eq. (2) is used in [15] in an electric distribution engineering application. The relationship between payback period Y and NPV is simply calculated for the case that the discount factor is zero (static economics, $\delta_k = 0$),

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$$NPV = (\frac{1}{Y} - 1)\Delta T \sum_{k=0}^{N} C_k \quad (3)$$

$$Y = \frac{\Delta T \sum_{k=0}^{N} C_k}{NPV + \Delta T \sum_{k=0}^{N} C_k}. \quad (4)$$

There have been cost/benefit calculation applications in distribution engineering, e.g., [16, 17]. But these applications are largely confined to deterministic formulations. Uncertainty in cost or benefit data could be assessed by allowing these input data to take on extreme values. Sensitivity of the payback period or the net present worth (over *N* project intervals) can be assessed fairly simply by varying input data. But in practical circumstances, there may be a large number of assumed input parameters, and it may be unreasonable to assume that several of these data take on extremal values simultaneously. Although it may be difficult to accurately assess probability models of the costs, benefit, and annual discount factor (e.g., 'cost of money'), these models may be estimated from historical data. The issues of probabilistic calculations in a power marketing environment are discussed in [18].

The use of net present value, and payback time in power distribution expansion planning are not the only tools in common use in this field. The literature documents many commonly used expansion planning methods, e.g., [19]. In most of these methods, assessment of costs and benefits are assumed to be deterministic.

II. Statistical models of project costs and benefits in distribution engineering

(A) Monetized values of costs and benefits

Monetized values of assets and the benefits accrued from the use of those assets are fraught with uncertainty. Perhaps the greatest source of this uncertainty is due to inflation and the concomitant value of money. In applications in which innovative concepts are applied to engineering projects, there may be uncertainty introduced to the economic assessment due to the fact that certain components are not yet fully in the commercialized sector. This is the case in power distribution applications.

The role of inflation in the uncertainty of power distribution components deserves special attention since inflation has been historically quantified and documented in the open literature, and this phenomenon has a temporal characteristic that is easily modeled. The Consumers Price Index (CPI) is often used as a weighted measure of the value of a given national currency, e.g., the U. S. dollar. References [20, 21] are a sampling of the literature on the calculation of the CPI. As an example, the CPI for the United States indexed to the average CPI for the year 2000 is shown in Fig. 1. Plotted in Fig. 1 is the CPI_{2000},

$$CPI_{2000} = \frac{CPI_{actual}}{CPI_{in\ year\ 2000}}.$$

By a least squares fit of CPI_{2000} versus the calendar year, y, one finds

$$CPI_{2000} = 1 + 0.02469(y - 2000). \quad (5)$$

The data for the least squares fit are from [22] for 1995 - 2016. Also depicted in Fig. 1 is a projection of the CPI to 2020 [23].

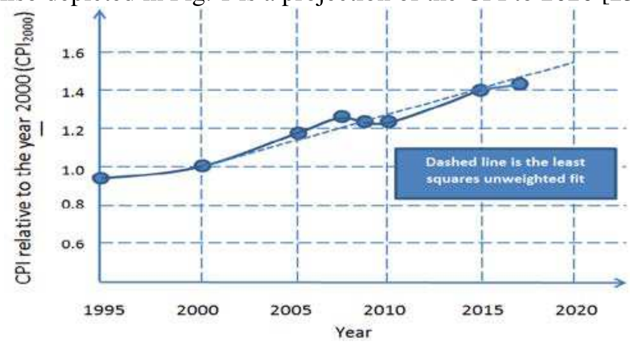

Fig. 1 The normalized Consumer Price Index of the United States (base year 2000, U.S. data abstracted from [22, 23], the 2017 – 2020 data are projected).

The Producer Price Index (PPI) is a similar measure and this index captures the average change over time in the commercial selling prices received by domestic manufacturers for their products. According to [22], "the prices included in the PPI are from the first commercial transaction for many products and some services." Samples of the PPI for electric power equipment and electric energy in the United States are shown in Fig. 2. The PPI is generally available categorized by commodity codes (e.g., commercial energy is coded 05-42, industrial energy is 05-43, heavy electrical equipment such as power transformers and voltage regulators is code 11-74). Since 2014, the categories for heavy electrical equipment have been refined and reorganized somewhat for the United States PPI tabulation. An observation from Fig. 2 is that a simple formulation as in (5) is probably not possible for most categories of commodities in the PPI; nonetheless, it is possible to use the PPI data for each year cited in commercial literature and compendiums of prices to normalize costs. This is done to obtain 'constant dollars'. For purposes of a cost / benefit analysis, whether using a probabilistic formulation or a deterministic calculation, it appears useful to normalize equipment cost prices by the PPI if the data are available, or by the CPI if the PPI data are unavailable.

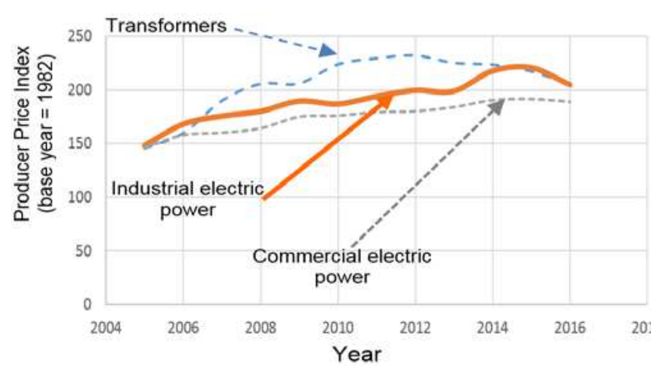

Fig. 2 The Producer Price Index in the United States for electrical equipment and services (normalized by the PPI in cited categories in 1982, shown in percent, data abstracted from [22]).

It is possible to enhance analysis accuracy by indexing asset costs to a given year, e.g., 2000. For actual distribution component data, it appears that working with a base year less than 20 years from the present gives very acceptable results: namely the cost / benefit analysis and actual evaluation of payback period agree within 2%.

The *costs* of key assets to be considered in a power distribution engineering application are (arranged approximately in order of economic significance, based on applications of future, innovative designs [24]):

- Substation transformers
- Distribution transformers (especially those of innovative design such as solid state transformers [25, 26])
- Protective devices, mainly fuses and circuit breakers (innovative designs of these devices have been proposed with high interruption speeds suitable for protection of solid state system components [27, 28])
- Lightning and surge protection devices
- Voltage regulators
- Shunt capacitors
- Distribution system hardware
- Conductors and associated hardware (1φ and 3φ)
- Maintenance for all assets over $0 \le t \le (N+1)\Delta T$.

Similarly, the key *benefits* that need to be monetized in a distribution engineering application include:

- Reliability – improved outage performance
- Accommodation of distributed energy resources
- Active power loss reduction and power factor correction requirements
- Improved (and automatic) voltage regulation and three phase balance
- Accommodation of energy storage devices
- Achieving the basic impulse level (lightning impulse insulation level) requirements.

(B) Probabilistic models

In the absence of a clear physical process which produces a given variate parameter, the selection of an accurate model of a probabilistic process may be a challenge. There are several methods that are in common use, however; but it seems that several of these methods are more motivated by mathematical convenience rather than physical accuracy. Perhaps the simplest of these models is the selection of a uniform probability density function (pdf) in which the pdf of a given variate parameter x is modelled as $f_x(x)$,

$$f_x(x) = \begin{cases} 1/(x_{max} - x_{min}) & x_{min} \le x \le x_{max} \\ 0 & otherwise \end{cases} \tag{6}$$

where x_{min} and x_{max} are the assumed extreme values of x, e.g., as obtained from a 'tornado diagram' [29]. For cases in which there is no physical process identifiable for the value of x, for example a range of values for the per unit cost of distribution components over a large range of suppliers, (6) appears to be a reasonable model for $f_x(x)$. A similarly mathematical convenience is the selection of a normal density,

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \tag{7}$$

where μ_x and σ_x denote the assumed mean and variance of x. In unusual circumstances, it may be possible to assign a physical process to the calculation of parameter x, such as in the assessment of the failure rate of a component by a 'bathtub curve'. In the cited example, it may be possible to assign a well known pdf to model x (e.g., the Weibull pdf); however, in many studies relating to distribution system costs and benefits, this approach has not been observed to be of value because the Weibull pdf does not model historical cost data well, and even if the Weibull pdf is used, the subsequent mathematics leads to computationally costly numerical calculations. A similar comment applies to the use of a variety of techniques for the construction of a pdf using historically obtained samples of variate x or the statistical moments of the variate. For example, applications of Edgeworth [30] and the Cornish – Fisher methods [31] leads to computationally costly numerical solutions required to obtain good statistical fits. For a range of tests using distribution expansion projects as test beds, alternatives to (7) were evaluated using a uniform probability density function as well as triangular densities. It is found that the ultimate results in the evaluation of the NPV are very similar to those obtained using the normal density (7).

III. Solution techniques

(A) Monte Carlo solutions

The Monte Carlo methodology is a repeated solution of a given problem using sample values of variate parameters. In the application to the calculation of the pdf of NPV as shown in (1), the variate parameters are B_i, C_i, and δ_k for $i = 0, 1, 2, \ldots, N$ and $k = 1, 2, \ldots, N$, thus $3N+2$ variates. The variates are pseudorandomly generated, often using a uniform or normal pseudorandom number generator feeding an appropriate algorithmic filter to obtain the desired (assumed) variate pdfs. The general approach is widely documented (e.g., [32, 33]). The main drawbacks of Monte Carlo simulations are the required number of sample solutions, the accuracy of generating the pseudorandom variates, and the fact that the simulation gives numerical results rather than literal, system theoretic results. None of these drawbacks have been found to be problematic in applications to new power distribution system designs. For example, in a project of ten years in which the resolution of the variates in (1) is one year, N is 9, and the number of degrees of freedom of the simulation is $3N + 2 = 29$. It is true, however, that the individual elements of B and C may be of high dimension. The approach is taken that many of the component costs in vector C may be grouped (added) and treated as a single variate. If some of the costs are distinctive and separate from others, these separate costs may be treated as separate variates with their own pdf. The same approach is used for elements of vector B. The initial costs and benefits (C_o, B_o) are separated in (1) due to their distinctive characteristics. By this approach, it is found that in distribution system NPV calculations, even using one million samples, a Monte Carlo analysis is readily accomplished on a laptop computer in a few seconds. The use of past CPI and PPI data are useful to render all the past cost and benefit data on a common dollar base, for example on a year 2000 base.

(B) System theoretic solutions

An alternative to the Monte Carlo approach is the use of system theoretic solutions to find the pdf of NPV from the densities of B, C, and δ. This approach becomes the solution of the calculation of a pdf of a function of several random variables. The approach has been suggested in [15] for the calculation of the pdf of a ratio distribution (e.g., (2)). When the variate pdfs of C and B are simple, this approach is valid (e.g., the calculation of the pdf of the ratio of two correlated or uncorrelated gaussian random variables, [34, 35]). Another approach is to convert the ratio distributions in (1) and (2) to some other mathematical form (e.g., the Mellin transform can be used to convert a ratio into a product which is more convenient in evaluations of probability density functions [36]). However, this approach too leads to a numerical solution to obtain the inverse Mellin transform which may not be particularly convenient.

IV. An Example Application in Power Distribution Engineering

(A) Description of the application: a next generation power distribution system

An example application is offered to illustrate the capabilities of probabilistic evaluation of net present value of an electric power distribution system based on the Kaldor-Hicks compensation principle. The basic configuration of a proposed 'next generation' power distribution system application is shown in Fig. 3. This system utilizes solid state distribution transformers (SSTs) at points of common coupling, and these transformers are rated 45 to 75 kVA. Solid state circuit interruption is used throughout the example system in the form of high speed fault interruption devices (FIDs). References [24 – 28] are a few of the growing number of descriptions of these solid state, power electronic devices for use in power distribution engineering.

The reason for using high speed interruption is twofold: to protect the solid state transformers downstream, and to isolate faults so rapidly as to obviate a load interruption (and hence improve the system average interruption frequency and duration indices, SAIFI and SAIDI). The example application shown here is the evaluation of five alternative designs listed in Table I. Note that maintenance is considered in an integrated form in C_i – a topic discussed in detail in [37, 38, 39]. Scheduling of maintenance [40] is not considered in this example, but maintenance costs *are* included in C_i.

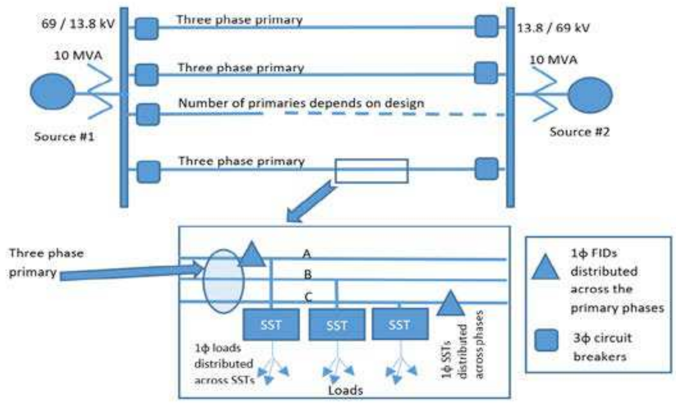

Fig. 3 The basic configuration of a doubly fed power distribution system. Single phase SSTs serve a number of loads. Fault interruption devices (FIDs) are distributed across the 13.8 kV primaries. The SSTs are assumed to be connected phase-neutral on the high side.

TABLE I FIVE ALTERNATIVE DESIGNS: AN EXAMPLE APPLICATION OF PROBABILISTIC COST TO BENEFIT EVALUATION

Design	Number of parallel primaries	Number of SSTs	Rating of SSTs (kVA, each)	Loads served per SST	Number of FIDs
A	3	222	45	3	37

TABLE II STATISTICAL PARAMETERS USED FOR AN EXAMPLE CALCULATION OF THE NET PRESENT VALUE OF FIVE ALTERNATE DESIGNS OF A 10 MVA POWER DISTRIBUTION SYSTEM

Design	Total cost figures mean (and S.D.) shown in dollars*10^6					Benefit figures mean (and S.D.) shown in dollars*10^6				
	C_0	C_1	C_2	C_3	C_4	B_0	B_1	B_2	B_3	B_4
A	7.076 (0.100)	0.7150 (0.0325)	0.8450 (0.0375)	0.9880 (0.0500)	1.1050 (0.0625)	0.010 (0.003)	7.00 (0.700)	7.00 (0.770)	7.00 (0.840)	7.00 (0.910)
B	7.076 (0.159)	0.7150 (0.0250)	0.7800 (0.0375)	0.8580 (0.0500)	1.1700 (0.0750)	0.035 (0.005)	9.00 (1.000)	9.00 (1.100)	9.00 (1.200)	9.00 (1.300)
C	7.497 (0.175)	0.5200 (0.0100)	0.5070 (0.0110)	0.5460 (0.0120)	0.7150 (0.0150)	0.028 (0.004)	6.50 (0.800)	6.50 (0.880)	6.50 (0.960)	6.50 (1.040)
D	7.497 (0.186)	0.4550 (0.2250)	0.4550 (0.0275)	0.5460 (0.0375)	0.7150 (0.0488)	0.039 (0.005)	7.50 (1.000)	7.50 (1.100)	7.50 (1.200)	7.50 (1.300)
E	7.497 (0.186)	0.4550 (0.2250)	0.4550 (0.0275)	0.5460 (0.0375)	0.7800 (0.0538)	0.056 (0.013)	9.50 (1.300)	9.50 (1.430)	9.50 (1.560)	9.50 (1.960)

Discount factor for each time interval i shown as mean values (and S.D. shown in parenthesis)

	$i = 1$	$i = 2$	$i = 3$	$i = 4$
δ_i	0.094 (0.004)	0.113 (0.005)	0.135 (0.009)	0.158 (0.012)

The example system in all designs is a 10 MVA, 13.8 kV primary distribution system with 400 single phase residential and light commercial secondary loads. Throughout the design a 60 kV Basic Impulse Level (BIL, also termed the Lightning Impulse Withstand Level) is accommodated [41]. The following elements are common to the five example designs and their analyses:

- This is a doubly fed system with two presumed separately derived sources, each rated 10 MVA.
- FIDs are used in the distribution primaries to isolate faults thus rendering high reliability, and the FIDs can interrupt 1255 A.
- The FIDs are all single phase devices in-line with the 13.8 kV primaries. The circuit breakers at feeder roots are conventional devices.
- The SSTs are controlled to effectuate unity power factor at the point of common coupling.
- The costs of the SSTs and FIDs are based on best available data from [22 - 24] and similar power electronic devices, mainly distribution class static var compensators.
- The SST and FID costs and benefits are grouped together for the parameters C_k, B_k, $k =$ 0, 1, …, 4.
- Statistical data for equipment costs are estimated by maximum and minimum estimates (e.g., [29]), using ±3σ as extremal values). For purposes of the probabilistic NPV evaluation, a pseudorandom normal probability density model (7) is used.
- Conductor sizes and costs are calculated appropriate to the number of service locations which are assumed to be uniformly located throughout the system. The longest feeder is 10 km.
- Reduction of system average interruption index and system average duration index has been monetized and considered in the benefits [42, 43].
- The benefit data B_i in Table II includes the avoidance of shunt capacitors needed for voltage support in a conventional distribution system; and these benefits also include the offset costs to attain at least 25% photovoltaic energy resources through the use of a DC port at the SSTs.

(B) The evaluation of expected NPV for five alternative designs

Table II summarizes the fixed and assumed probabilistic parameters of five alternative 10 MVA distribution system designs. In all cases, the time intervals indicated are taken to be two years in duration with the statistics shown in Table II stationary within each interval. For example purposes, each of the five designs are evaluated using nonsequential Monte Carlo simulations with 50,000 samples each. It is possible to track the probability that $NPV > 0$ by a coefficient of variation to ensure that this probability has 'converged'. This approach is shown in connection with the calculation of loss of load probability (LOLP) in [44].

For the cases illustrated, Excel spreadsheets were used to calculate the mean values and standard deviations from data downloaded from appropriate sources. Matlab was used to perform the Monte Carlo studies. For a system of this size and N = 4 (i.e., five time intervals), the processing time for a 50,000 sample study was a few seconds on a laptop computer. Most of this time is required to assemble the histograms of the results.

(C) Example results

The results of this example study may be represented in many ways. Figure 4 shows a tornado diagram [27] of the NPV for the five alternatives studied. Additional results are shown in Table III, and Figs. 5 and 6 show the histograms of NPV for designs {A,B} and {C, D, E} respectively. The ordinates in Figs. 5 and 6 are the number of samples out of 50,000 (total) in the indicated bins. Conversion of these axis numerical values to the pdf is attained by dividing by 50,000. In Table III, the skewness and the kurtosis (γ and κ respectively) are defined as,

$$\gamma = E[\left(\frac{NPV-\mu}{\sigma}\right)^3] \quad (8)$$

$$\kappa = \frac{E[(NPV-\mu)^4]}{(E[(NPV-\mu)^2])^2} \quad (9)$$

where E[.] denotes expectation, NPV are the sample net present values, μ is the mean of the NPV, and σ is the standard deviation of the NPV.

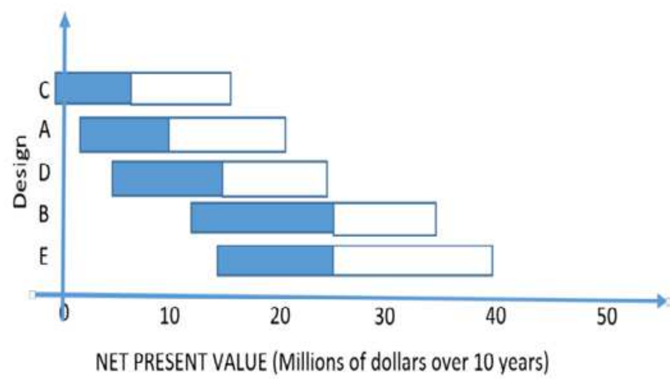

Fig. 4 Tornado diagram of net present value for five alternative designs studied in the example application. Ranges of NPV shown as the mean value ±2σ, for a ten year project period. The darkened portion is below the mean, the white portion above the mean.

TABLE III STATISTICAL RESULTS OF THE EXAMPLE FIVE DISTRIBUTION SYSTEM DESIGNS*

Design	Mean NPV (Millions of dollars)	S.D. of the NPV (Millions of dollars)	Probability of NPV > 0	Skewness of NPV (γ)	Kurtosis of NPV (κ)
A	11.84	4.44	0.9999	0.984	5.490
B	24.32	5.83	1.0000	1.022	5.799
C	8.19	4.25	0.9955	1.034	5.666
D	14.47	4.93	1.0000	1.031	5.673
E	26.79	6.32	1.0000	1.360	12.321

*Ten year project life assumed, 50,000 Monte Carlo samples used in each study

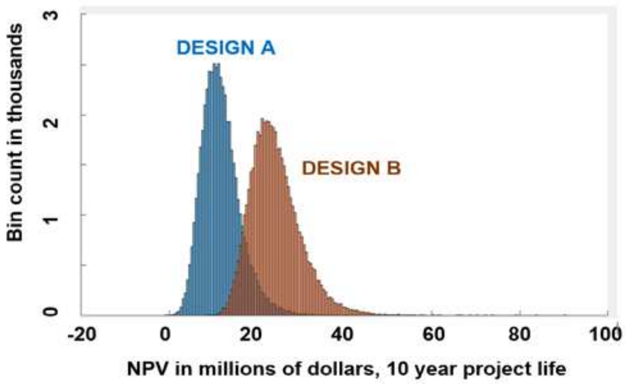

Fig. 5 Histograms of net present value for designs A and B in the illustrative example. The vertical scale is the number of NPV samples out of 50,000 in the Monte Carlo study. Thus the vertical scale is proportional to the probability density function of NPV. Design A has five Monte Carlo samples which fail the Kaldor-Hicks criterion (out of 50,000).

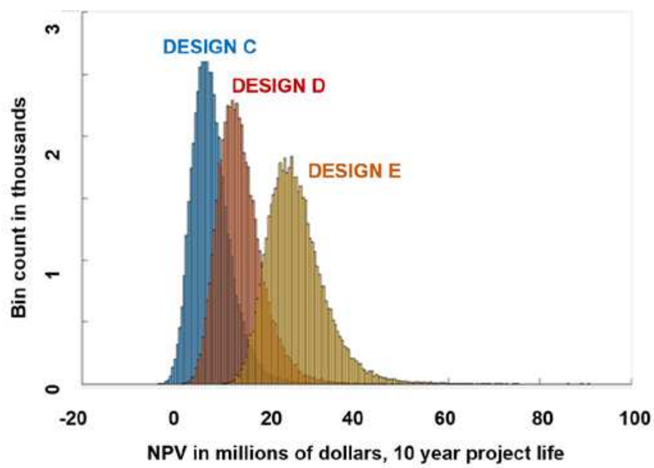

Fig. 6 Histograms of net present value for designs C, D and E in the illustrative example. The vertical scale is the number of NPV samples in each histogram bin out of 50,000 in the Monte Carlo study. Thus the vertical scale is proportional to the probability density function of NPV. Design E is the configuration with five three-phase primaries and the largest number of services per SST (namely five such secondary services). Design C exhibits 225 Monte Carlo samples which fail the Kaldor-Hicks criterion (out of 50,000).

Skewness is a measure of the asymmetry of the histogram of the NPV. In each of the designs studied, the right tail of the histogram of NPV is longer than the left tail, and this condition is termed *positive skew*. As verified by the entries in Table III, all skewness figures are positive, and the highest skewness is shown in designs A and E.

The kurtosis is a measure of the number and extent of outliers. Because of the use of the fourth power in the definition of κ, large outlier values of NPV contribute greatly to κ, and therefore the kurtosis is a measure of the impact and extent of the extremal values of NPV. Design E shows the largest number and highest values of positive outliers and this is reflected in the highest kurtosis in Table III. Note that the kurtosis of a standard normal distribution is exactly 3.000, and it is clear that the probability densities observed in this example contain more outliers than expected for the case of standard normal variates. Visual inspection of the histograms obtained in the example suggests that the probability distributions may resemble the Weibull distribution (for which κ is typically above 4.0). Both skewness and kurtosis have been criticized for having values that are difficult to interpret (e.g., [45]), but in this application, the positive skew and presence of positive outliers appear to favor some designs (e.g., design E in this case). In this example, the kurtosis of the NPV in designs A, B, C, and D consistently increase with the number of samples in the Monte Carlo simulation; design E exhibits high valued outlier NPV levels, and the kurtosis generally increases with the number of samples, but not monotonically. Extreme values have been discussed in power distribution engineering contexts, e.g., [46]. No negative outlier values of NPV are observed in design E.

(D) Discussion of alternatives

Using only the Kaldor-Hicks compensation criterion, NPV > 0, design E described above appears to be a favored choice of the five alternatives. The mean NPV is the highest in design E and the skewness and kurtosis appear to favor higher values of NPV in the Monte Carlo simulations. Design E utilizes the larger number of FIDs and parallel distribution primaries. The former results in high reliability levels due to the efficient interruption of faults, and the latter results in lower circuit losses and dispersion of loads across a larger number of distribution primaries. Design E unfortunately exhibits high risk from the point of view of the highest investment.

It is concluded that the probabilistic assessment of NPV and the Kaldor-Hicks criterion, the probabilistic assessment is of some value in making a decision on implementing the distribution design. However, the assessment and approach shown here is not the only tool for decision making. Surely conventional assessment tools from classical distribution engineering, and a range of considerations such as evaluation of the accuracy of the cost and benefit data used, motivation to pursue new 'smart grid' type designs, compliance with national, regional, and company norms should be included. A main point of the probabilistic approach to capture the NPV is that if the statistics of project cost and benefit data can be estimated, it appears useful to use those data to obtain the statistics of the NPV. Arguably, the statistics of NPV give a potentially more realistic assessment of NPV as opposed to deterministic calculations or calculations that use extremal values of C_k and B_k.

V. Pitfalls of the Kaldor – Hicks Compensation Principle

As indicated above, the Kaldor – Hicks principle is essentially a straightforward analysis of the discounted cost benefit flow of a project over its expected life. The approach is different from Pareto analysis of a multiobjective problem (e.g., [47, 48]) because the Kaldor-Hicks compensation permits the improvement of some project objectives at the expense of others whereas the essence of Pareto optimization is that no project objective can be improved without the worsening of another. It is unclear whether this attribute of the Kaldor-Hicks approach is a shortcoming, but it is clear that Kaldor-Hicks is only one of a number of tools in assessing the economic efficiency, the feasibility, and the engineering acceptance of a project. Pitfalls of the Kaldor-Hicks approach include the following:

- Unmeasurable (or inestimable) uncertainties might not be accounted (e.g., required safety measures)
- Postponements and delays of project segments are difficult to incorporate
- Reversal or cancellation of project segments are difficult to incorporate
- Changing governmental and regulatory requirements may be difficult to model probabilistically.

In many distribution engineering projects, particularly for the design and implementation of entire systems, risk aversion is often practiced to favor economically successful projects. Risk aversion is nominally accounted by overestimating costs. Or, in the application cited in this paper, the pdfs of costs and/or benefits might be widened. For distribution engineering applications, some form of *stress testing* might be applied, and the examination of extreme cases that correspond to low NPV might be examined carefully.

VI. CONCLUSIONS

This paper describes a reformulation of the calculation of the net present worth of an electric power distribution system as a probabilistic problem. The procedure uses historical data for cost and benefit data as obtained from public sources, engineering documents, and past projects. The historical data are used to formulate an approximate probability density of costs, benefits, and discount factors for the expected lifetime of the project. In this way, uncertainty in project economics is captured rather than ignored. The concept is to render the Kaldor-Hicks criterion of positive valued NPV in the form of the *probability* of NPV > 0; and to calculate other statistics of the variate NPV (e.g., mean value, standard deviation, and the conditional expectation of NPV given that NPV < 0). These statistics give information on the *economic efficiency* of a projected infrastructural improvement or new power distribution engineering project.

The shortcoming of the probabilistic approach described is the accuracy of available data on component costs and calculated benefits. Also, it is noted that the application of the Kaldor-Hicks compensation principle is only one indicator of economic efficiency and engineering feasibility. This approach does not capture many other safety, political, regulatory and public acceptance issues.

An example is given to illustrate the probabilistic approach, and how the statistics of NPV can be calculated as applied to the design of a new generation power distribution system. Since innovative new designs have considerable cost and benefit uncertainty, the probabilistic calculation of NPV appears to be useful to capture and quantify that uncertainty.

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