Sparse Optimal Control of LTI Systems Under Sparsity-Dependent Delays

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Background

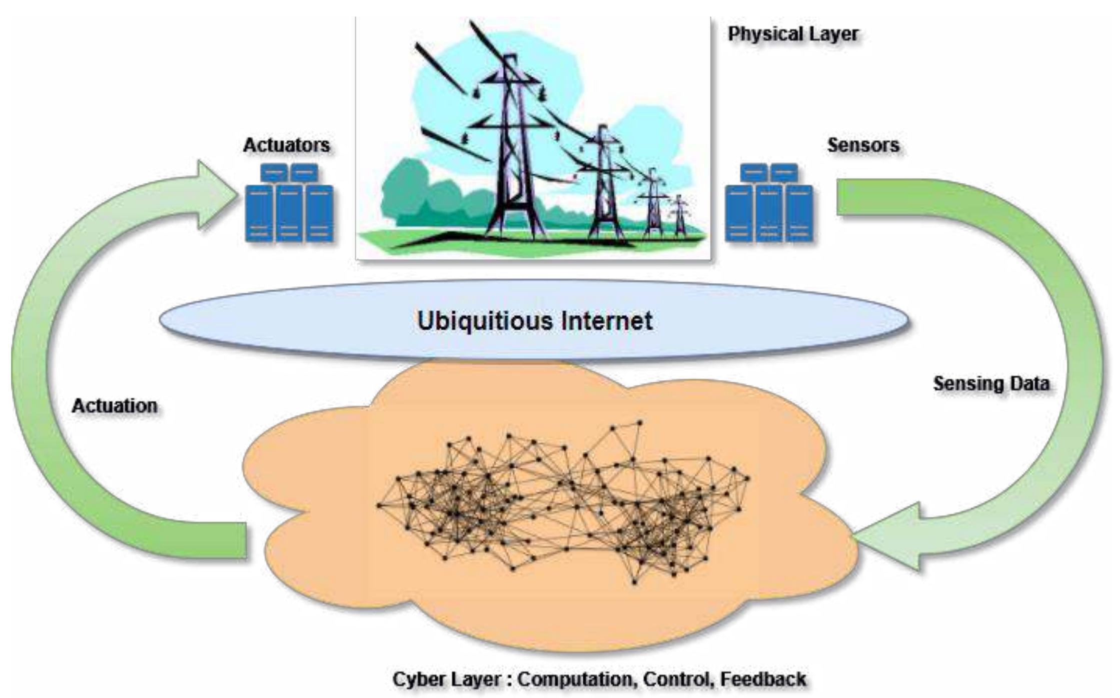
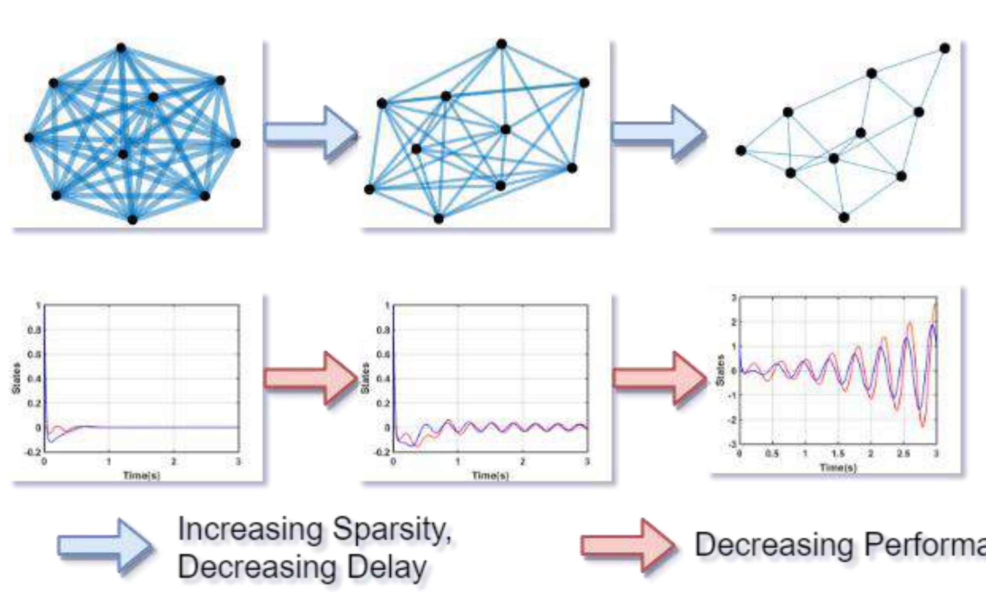


Fig: A Simplified Cyber-Physical System

- Integration of computation, networking and physical process.
- τ is the feedback delay. Control requires all-to-all communication.
- Result – Dense Communication network, Higher Infrastructure costs.
- Solution – 'Sparsify' the communication network. Gain matrix $K_{ij} \to 0$.
- Limited Bandwidth – leads to presence of feedback delays.
- Delay is a function of sparsity.
- Trade-off : Increasing sparsity - degrades performance - decreases delay - improves performance

Problem Statement



- Closed loop delayed System:

$$\dot{x}(t) = Ax(t) + BKx(t-\tau) + B_w w(t) \quad (1)$$

$$z(t) = [Q^{\frac{1}{2}} \quad R^{\frac{1}{2}}K]^T x(t) = C\,x(t).$$

- Let K stabilize the delayed system for τ.
- AIM – three objectives :

  Delay Stable, Optimal H2 norm, Sparsify K.
- H2 Problem :

Find proper, rational K to internally stabilize (1) and minimize H2 norm of transfer function from disturbance $w(t)$ to output $z(t)$ for $\tau = 0$.

Algorithms Developed

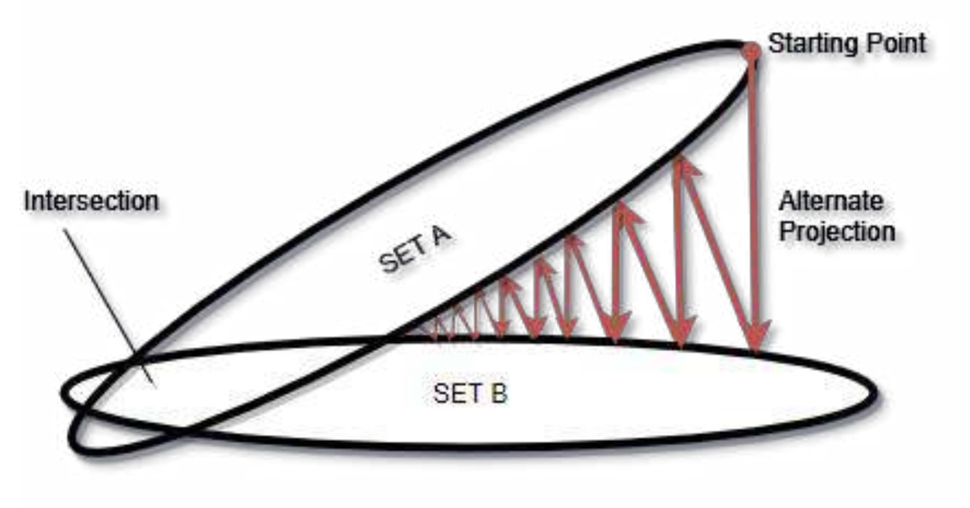
- $J(F)$ is the H2 norm of F with delay τ. $g(K)$ is the weighted l_1 norm of K.

$$\text{minimize} \quad \overbrace{J(F)}^{\mathcal{H}_2} + \gamma \overbrace{g(K)}^{\text{Sparsity}}$$

$$\text{subject to } F = K$$

$$C_{\mathrm{BMI}_1}(K,\tau) \succ 0, \; C_{\mathrm{LMI}_2}(\tau) \succ 0 \} \rightarrow \text{Delay Stability}$$

- Alternating Directions Method of Multipliers (ADMM) steps:



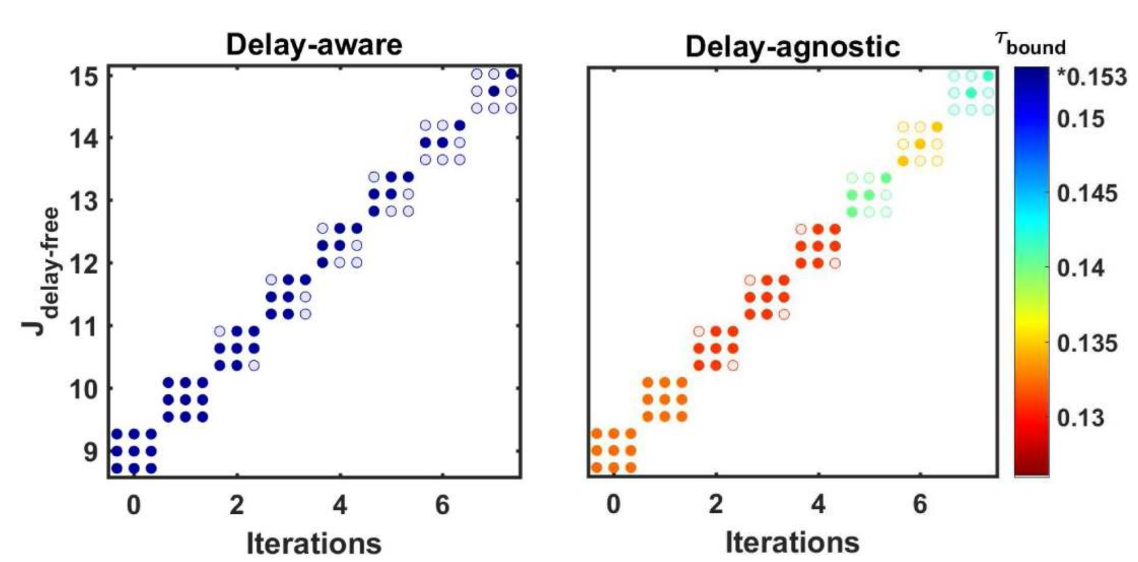
- Set A : K that satisfy delayed H2 optimality problem.
- Set B : Sparse K that are stabilizing for (1).

- Alternatively solve each objective while maintaining stability with respect to delay.
- Delay Vs Sparsity – As sparsity increases, delay decreases.
- τ dependency is on card(K). Cannot be explicitly included in ADMM.
- As sparsity increases, τ decreases $\Longrightarrow K$ needs to be stabilizing for a smaller delay $\Longrightarrow$ set of stabilizing solution expands.
- For optimal H2 norm - need to include τ.
- Delayed performance is bad, especially, in case of large τ value.
- Delayed Systems are of infinite dimensions - Discretize the interval $[0, \tau]$ and create an augmented vector.
- Augmented vector– Includes past states.
- Define $J_\tau(K)$ on the augmented vector.
- Positive and finite iff the system is stable with the feedback delay.
- $\mathcal{A}$ – new state matrix is defined to imitate the effect of delayed states.

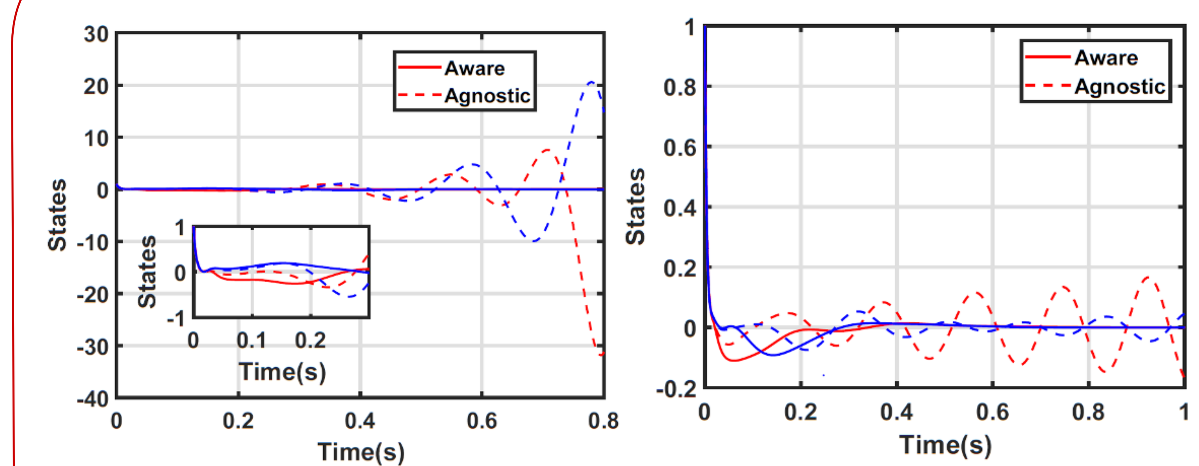
$$\eta = \begin{pmatrix} \phi(-\tau_N = -\tau) \\ \phi(-\tau_{N-1}) \\ \vdots \\ \phi(-\tau_2) \\ \phi(0) = x(t) \end{pmatrix} \Longrightarrow \mathcal{A}\eta = \mathcal{A}\begin{bmatrix} \Phi(-\tau) \\ \vdots \\ x(t) \end{bmatrix}$$

Simulation Results

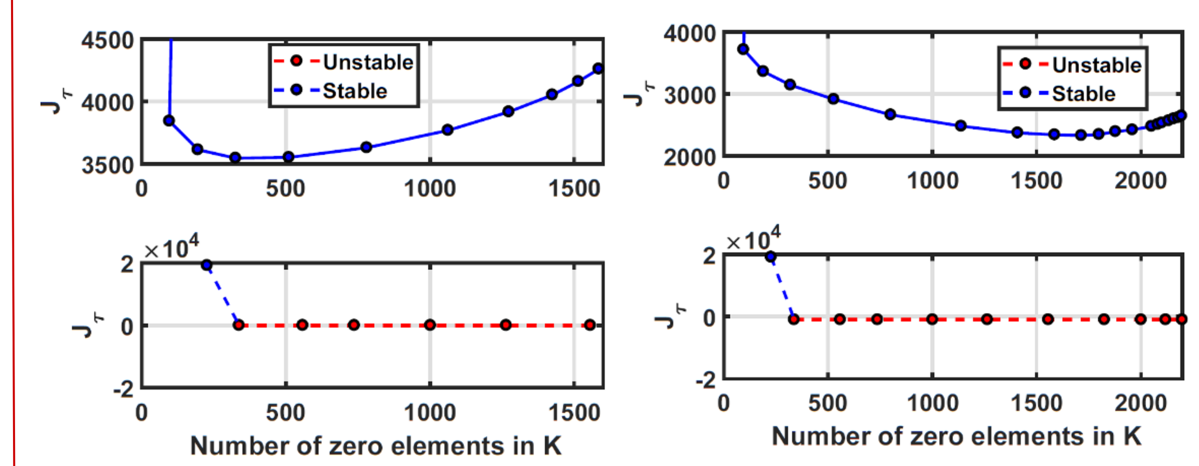
- Comparison with Delay-agnostic Algorithms.
- Case – 1 : K is a 3x3 matrix. Observe the consistent maintenance of delay-bounds in our delay-aware algorithm. This does not include optimality with respect to the delay.



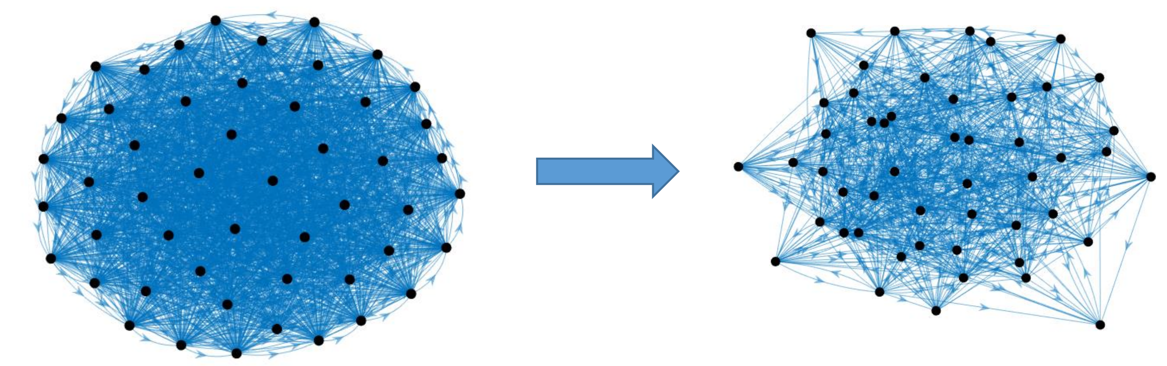
- Case – 2 : K is a 50x50 matrix consisting of 2500 links. We include delay in H2 norm for this case.



(a) Constant Delay (a) Variable Delay



(b) Constant Delay (b) Variable Delay



- The number of non-zero elements of K is decreased from 2500 to 924.q

Conclusion

- We design algorithms with convergence guarantees to find sparse optimal controllers that are delay stable.
- As part of future work, we expand the idea to real world multi-user system where each user is dynamically decoupled from each other and share a common cloud-based communication network.

References

N Negi and A Chakrabortty. ACC 2018.
" Sparse Optimal Control of LTI Systems under Sparsity-Dependent Delays".