Method to Minimize Space Harmonics of Fractional Slot Concentrated Windings in AC Machines

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Abstract- A winding method for AC Machines is presented to eliminate both the sub and super space harmonics of fractional slot concentrated windings where the stator slot numbers are integer multiples of six. The proposed method is based on three single-layer concentrated windings where the number of turns in one layer is optimized whereas the other layers are shifted to cancel the sub-harmonics. The cancellation of super-harmonics is achieved through series connection of two space shifted layers which effectively doubles the number of slots. The proposed method has been successfully validated in widely used 12-slot/10-pole and 18-slot/14-pole machine configurations. Applying the proposed winding concept to a Permanent Magnet Synchronous Machine (PMSM) demonstrates the cancellation of dominant sub and super harmonics, THD reduction, torque ripple reduction, and core loss minimization.

I. INTRODUCTION

PMSMs with fractional slot concentrated windings (FSCW) where slot/pole/phase is less than one are gaining interest in hybrid and electric vehicles [1], aerospace [2], and white goods applications. Commonly, FSCW offers two different winding structures. The non-overlapping teeth windings are called the single layer concentrated windings (SLCW) and around the teeth windings are known as the double layer concentrated windings (DLCW) based on the number of layers they create on each slot. In [3], the performance of FSCW and distributed windings are compared where it has been found that the FSCW windings have much lower copper loss due to the lower short end winding and high fill factor. Additionally, FSCW has other advantages of high power density, wide speed range [4], low cogging torque, fault tolerance, and low manufacturing cost [5]. However, FSCW does not provide sinusoidally distributed stator magnetomotive force (MMF) due to low slot/pole/phase. Therefore, the magnetic field produced by these windings contains several higher (super) and lower (sub) order harmonics along with the torque producing component. These super and sub harmonics rotate asynchronously with respect to the rotor are generally responsible for additional eddy current loss in the permanent magnets (PM) and the windings itself, and core loss in both stator and rotor. The associated eddy current loss in PM increases its temperature and will eventually increase the possibility of PM demagnetization [6]. Moreover, these undesirable space harmonics also lead to increased noise and vibration [7].

To improve the stator MMF quality and performances of the FSCW regarding eddy current loss, PM loss, noise and vibrations, the cancellation or reduction of the super and sub harmonics is crucial. Several methods and techniques have been developed in the past to improve the MMF of FSCW. In [8], a combined wye-delta connection has been used to cancel the sub-harmonics. However, the wye-delta concept can completely cancel the sub-harmonic when the slot number is a multiple of twelve. Additionally, the super harmonics still exist in the MMF spectrum and present problems in terms of rotor losses, noise, and vibration. In [9], a generic winding approach has been used to cancel the sub-harmonics utilizing three sets of windings shifted in both space and time. However, it requires three different sets of inverter to create time shift and at the same time, dominant unwanted super harmonics still exist in the stator MMF. However, multiple inverters bring more complexity in the controller and are not economically viable. In [10], a four-layer winding has been developed from DLCW merging the coil groups under the same phase and slot to cancel the sub-harmonics. However, this method is only applicable to the slot/pole combinations where the condition of merging the same phase coils in the same slot can be satisfied. Additionally, the fundamental winding factor has been reduced by more than 3.5% and all the dominant super harmonics still exist. In [11], two winding sets have been created which are shifted in space to cancel one particular super harmonics of the stator MMF of FSCW. However, in doing so, the other dominant sub-harmonics still exist in the MMF spectrum. In [12], two-sets of three phase space shifted wye-delta windings have been proposed to cancel simultaneously both the super and sub harmonics from the MMF. However, this method is effective when the number of slot is multiple of twelve.

In this paper, a method is proposed to cancel the dominant super and sub harmonics of FSCW where the slot numbers are integer multiples of six. The method is based on three single layer concentrated windings having two three-phase winding sets shifted in space. The turn number of first layer is multiple of the other layers whereas the second and third layers are shifted having reverse polarity with respect to the first layer for both the winding sets. The proposed winding method is applied to the widely used 12-slot/10-pole and 18-

slot/14-pole PMSM to prove the effectiveness of the method. The application of the winding in PM machines can effectively cancel the sub and super harmonics at the same time while reducing the core loss and the torque ripple at the expense of small reduction of the fundamental winding factor.

PROPOSED WINDING CONCEPT

The proposed winding concept is based on three SLCW coils to cancel the sub and super harmonics of the FSCW. Each stator slot is divided into three segments to adopt three single layer concentrated winding structure as shown in Fig. 1. The first SLCW has a turn number of N_1 and is placed in the first layer of the slot. Second SLCW of turn number N_2 is shifted by one slot pitch on the left with reverse polarity with respect to the first SLCW. The third SLCW of turn number N_2 is shifted by one slot-pitch on the right with reverse polarity with respect to the first SLCW placed on the third layer. For the Qslot/P-pole configuration of FSCW, the torque-producing component is the $(P/2)^{th}$ space harmonics while the other space harmonics create unwanted noise, torque ripple, vibrations etc. The MMF vector diagram of the k^{th} harmonics is presented in Fig. 2(a). The angular difference between 1st SLCW and 2^{nd} SLCW is $k \cdot 2\pi/Q$ and between 1^{st} SLCW and 3^{rd} SLCW is also $k \cdot 2\pi/Q$, where k is the harmonics order. The effective number of turns of k^{th} harmonics can be evaluated as

$$N_1 - 2N_2 cos\left(\frac{k \cdot 2\pi}{Q}\right)$$
(1)

where Q is slot number, and N_1 and N_2 are the turn numbers for different layers. It is evident from (1) that through proper selection of the number of turns on the first layer, any dominant harmonic can be cancelled. This proposed method of canceling sub-harmonics of FSCW is applicable when the slot number is multiple of six (6, 12, 18, 24, 30, and so on). The resultant MMF vector diagram for the torque-producing component is shown in the Fig. 2(b).

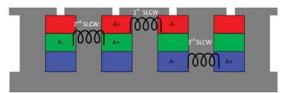


Fig. 1: Three Layer Concentrated Winding

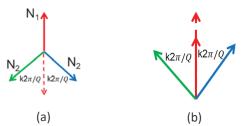
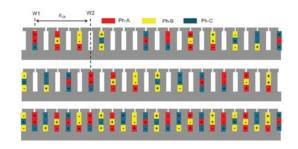


Fig. 2: MMF vector diagram of proposed winding (a) MMF of kth harmonics (b) MMF of Torque producing harmonics.

In order to cancel a dominant super harmonics, two three-phase winding sets W1 and W2 are created based on the previously mentioned three single-layer concentrated windings by doubling the number of stator slots. W1 and W2 are space shifted by θ_{sh} and connected in series as illustrated in Fig. 3. The proper selection of the shifting angle can cancel one dominant super harmonics. Therefore, the proposed method can cancel simultaneously both the super and the sub harmonics providing a cleaner spectrum compared to the conventional DLCW and SLCW. To analyze the proposed concept, a widely used 12-slot/10-pole configuration is used to develop the winding function equation. However, the proposed concept of harmonic reduction can be applied to any FSCW where the slot numbers are integer multiples of six. The MMF distributions of the two winding sets are shown in Fig. 4. The winding function for winding set W₁ is



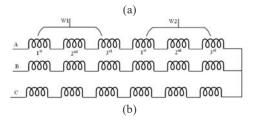


Fig. 3: (a) Proposed winding concept, (b) Three phase connection

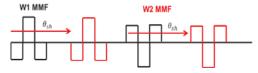


Fig. 4: MMF distribution of proposed winding

The corresponding air gap MMF of the three layers of winding set W1 is

$$MMF_{W1_1} =$$

$$\sum_{k=1,-5,7,\dots}^{\infty} \frac{6N_1I}{k\pi} \sin\left(\frac{k\pi}{12}\right) \cos\left(k\left(\theta - \frac{\pi}{6}\right) - \omega t\right) \dots \dots \dots (5)$$

$$MMF_{W1_2} = -\sum_{k=1,-5,7,\dots}^{\infty} \frac{6N_2l}{k\pi} sin\left(\frac{k\pi}{12}\right) cos(k\theta - \omega t) \dots \dots \dots \dots (6)$$

$$\begin{split} &MMF_{W1_3} = \\ &- \sum_{k=1,-5,7,\dots}^{\infty} \frac{6N_2I}{k\pi} sin\left(\frac{k\pi}{12}\right) cos\left(k\left(\theta - \frac{2\pi}{6}\right) - \omega t\right) \, \dots \dots \dots (7) \end{split}$$

Here, I is the peak current in the winding, ω is the angular frequency, and θ is the space angle. The air gap MMF of the winding W1 can be expressed as

Similarly, the air gap MMF of the winding W2 can be expressed as

The total air gap MMF of the winding of the proposed concept is given by (10) as

It is evident from (10) that $N_1 - 2N_2 \cos\left(k\frac{2\pi}{12}\right)$ can be tuned to cancel the sub-harmonics of FSCW and at the same time, $\cos\left(\frac{k\theta_{sh}}{2}\right)$ can be made zero to cancel one dominant super harmonic by selecting, $\cos\left(\frac{k\theta_{sh}}{2}\right)$ or, $\theta_{sh} = \frac{3\pi}{k}$ or $\frac{\pi}{k}$. For a 12-slot/10-pole machine, $N_1 = 2N_2 \cos\left(\frac{2\pi}{12}\right) = \sqrt{3}N_2$ can cancel the sub harmonics, k = 1. However, the dominant super harmonics 7^{th} space harmonics can be canceled using $\theta_{sh} = 77.1$ degree or $5\alpha_s$, α_s is the slot pitch.

Moreover, for a machine with slot number Q, the turn number of the first layer will be selected based on $N_1=2N_2\cos\left(k\cdot\frac{2\pi}{Q}\right)$ to cancel a dominant sub-harmonic or a super harmonic. Additionally, $\theta_{sh}=\frac{3\pi}{k}$ or $\frac{n\pi}{k}$, (n odd multiple of π) is selected to cancel one dominant super harmonic or sub-harmonic. In Table I, the turn number and shifting angle θ_{sh} are summarized for several slot/pole combinations to show the effectiveness of the proposed winding method. This proposed concept is not limited to only the slot/pole combinations of Table I rather; it is effective for any FSCW where slot number is a multiple of six.

For a 12-slot/10-pole configuration, based on the turn number and shifting angle, the harmonic spectrum of stator MMF, obtained analytically, is shown in Fig. 5. Observing the stator MMF spectrum of conventional DLCW and the proposed concept, it is evident that the latter can effectively cancel 1st and 7th order space harmonics. Additionally, it also reduces 17th order space harmonics by more than 60%. However, the fundamental winding factor will be reduced by approximately 5%.

For an 18-slot/14-pole configuration, the proposed method can also effectively cancel unwanted sub and super harmonics. In 18-slot/14-pole configuration, due to the low machine periodicity (t = GCD(Q, P/2)), both odd and even harmonics exist in SLCW while only odd harmonics exist in DLCW. However, the proposed concept is able to cancel most of the undesired harmonics and shows a cleaner MMF spectrum as shown in Fig. 6. The 1st and 11th order harmonics have been effectively cancelled, and 5th order space harmonic is reduced to 50% of the original. To achieve this cleaner MMF spectrum, the fundamental winding factor is also reduced by 9%.

Therefore, the proposed winding method can cancel most of the unwanted space harmonics for different slot/pole combinations where the slot number is a multiple of six. The cancellation of all these unwanted space harmonics comes at an expense of a small reduction in the fundamental winding factor

TABLE I SHIFTING ANGLE AND TURN NUMBER FOR DIFFERENT SLOT/POLE CONFIGURATIONS

Q/P	$k < \frac{P}{2}$	$\frac{N_1}{N_2}$	$k > \frac{P}{2}$	θ_{sh} (degree)	$\frac{Q}{P}$ (New)
12/10	1	$\sqrt{3}$	7	77.15	24/10
12/8	2	1	8	67.5	24/8
18/14	1	1.9	11	49	36/14
24/16	4	1	16	33.5	48/16

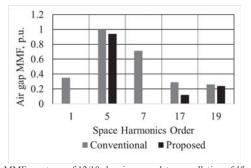


Fig. 5: MMF spectrum of 12/10 showing complete cancellation of 1^{st} and 7^{th} order harmonics where 5^{th} is the torque producing component

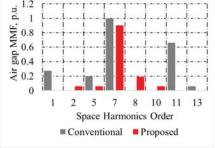


Fig. 6: MMF spectrum of 18-slot/14-pole machine.

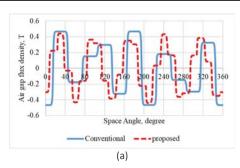
III. PERFORMANCE ANALYSIS

To validate the proposed winding method, two pairs of PM machines have been designed through FEA, namely; (i) 12slot/10-pole with conventional DL windings and 24-slot/10pole with proposed three-layer winding, (ii) 18-slot/14-pole with conventional DL winding and 36-slot/14-pole with the proposed three-layer winding. Additionally, proposed concept doubles the stator slot number compared to the conventional design, such as, 12-slot/10-pole becomes 24-slot/10-pole. Different slot/pole combinations have been selected to demonstrate the universality of the proposed concept. However, both the 12-slot/10-pole [8,10,11] and 18-slot/14pole [9,13] have widely been used by the researchers to show improvement of harmonic spectrum in FSCW concept. Same magnet dimensions, rotor dimensions, phase currents, DC link voltage, and outer dimensions have been assumed for the DL and the proposed windings with the same pole numbers. The key design parameters for the machines are given in Table II.

The air-gap flux density and corresponding MMF spectrum due to stator excitation derived from 2D finite element analysis (FEA) with current mode simulation have been analyzed next. A solid rotor with point current source has been used to avoid any contribution from the rotor and slotting.

TABLE II
DESIGN SPECIFICATIONS

Parameters	Values
Outer Diameter, D _{out}	145 mm
Axial Length, L	60 mm
Rated Power, P	1 kW
DC Voltage, V_{dc}	96 V
Speed, ω	1000 rpm



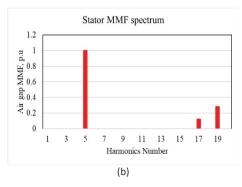


Fig. 7: (a) Air gap MMF of 12-slot/10-pole (conventional) and 24-slot/10-pole (proposed) windings, (b) Spectrum of proposed 24-slot/10-pole (FEA) due to stator excitation.

The air-gap flux density due to stator excitation for the proposed 24-slot/10-pole and conventional 12-slot/10-pole are shown in the Fig. 7(a) using the proposed turn numbers and shifting angle as calculated in Table I. The corresponding space harmonic spectrum of 24-slot/10-pole is shown in Fig. 7(b) and it is evident that the new winding method cancelled the dominant sub and super space harmonics. The 1st and 7th space harmonic is reduced by more than 60%. However, the 19th space harmonic remains the same. The FEA results match with those from the analytical calculations in section 2.

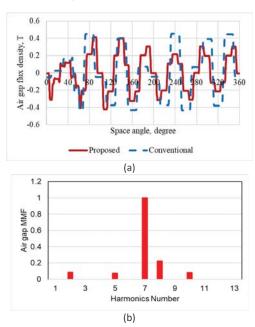


Fig. 8: (a) Air gap MMF of 18-slot/14-pole (conventional) and 36-slot/14-pole (proposed), (b) Spectrum of proposed 36-slot/14-pole (FEA) due to stator excitation.

The air-gap flux density due to stator MMF contribution is shown in Fig. 8 for the 36-slot/14-pole configuration using the proposed winding method. It is evident that the 1st and 11th orders are completely cancelled, and additionally 5th space harmonic is reduced by a substantial amount. However, the low machine periodicity introduces 2nd and 8th in the spectrum which are less than 20% with respect to the fundamental. The downside of the proposed concept for this PM machine is the reduction of the fundamental winding factor. The FEA results for both of the slot/pole configurations support the analytical representation of the new three-layer winding concept.

It is may not always be possible to get the expected turn ratio as the number of turns cannot be a fractional value. Table III and Table IV show the effect in the performances due to the variation in turn numbers. The required turn ratio from analytical expressions is $\sqrt{3}$ for 24-slot/10-pole, which makes the most desirable turn number of the first layer to be a noninteger. Therefore, a turns ratio of 1.83 can be achieved in order to make the turn number in the first layer to a realizable integer value. However, a turns ratio of 2 is chosen instead of the theoretical 1.9. Deviation from the required turns ratio increases the magnetic core loss and torque ripple by a small amount.

TABLE III REALIZABLE TURN RATIO FOR 24-SLOT/10-POLE

Design (24-slot/10- pole)	N_1/N_2	Core loss, W	Ripple, $T_{pk-pk}\%$
Theoretical turn ratio (1.73)	10.4/6	8.7	2
Realizable turn ratio (1.83)	11/6	8.8	2.1

TABLE IV REALIZABLE TURN RATIO FOR 36-SLOT/14-POLE

Design (36- slot/14-pole)	N_1/N_2	Core loss, W	Ripple, $T_{pk-pk}\%$
Theoretical turn ratio (1.88)	9.4/5	9.8	1.4
Realizable turn ratio (2.00)	10/5	9.9	1.5

Electromagnetic performance at rated speed and rated phase currents are shown in Table V. The average torque for 24-slot/10-pole machine using the proposed winding is 11.7 Nm compared to 11.9 Nm for the DLCW as illustrated in Fig. 9. One of the major sources of ripple contribution in the torque comes from the interaction of same order stator and rotor MMF harmonics. The corresponding improvement in torque ripple is 88% due to harmonic reduction and doubling of slot number. The torque ripple of the conventional design is 15% compared to 2% for the proposed concept. This improvement in ripple reduces the necessity of conventional pole shaping and step skewing which also helps to reduce manufacturing complexity. The overall magnetic loss (PM and core) reduction of the designed machine is 40%. The PM and rotor core loss using the proposed concept is 9.5 W compared to 14 W for the conventional winding. However, the overall average torque reduction in getting the cleaner MMF spectrum is 3% which is less than expected due to final selection of shifting angle.

TABLE V RATED PERFORMANCE OF 24-SLOT/10-POLE

Parameters	Conventional (12-slot/10-pole)	Proposed (24-slot/10-pole)
Torque, T_{avg}	11.9	11.7
$T_{Ripple}(\%)$	15	2.5
$V_{LL}(1^{st})$	90	88
$THD_{V_{LL}}(\%)$	4.7	3.7
$P_{core}(W)$	14	9.5
$P_{magnet}(W)$	6	1
Power factor	0.96	0.93

From Fig. 10, the average torque for the proposed 36-slot/14-pole is 12.7 Nm compared to 13.7 Nm due to the reduction of fundamental winding factor. However, the shifting angle of the three-phase winding sets had to be changed to 49.75° from the ideal 49° due to the teeth saturation and slotting effect from from the rotor itself. The peak-peak torque ripple improvement is 70% (2% compared

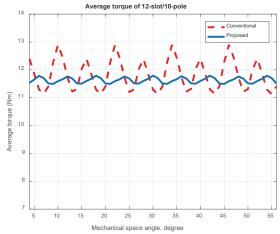


Fig. 9: Output torque for 10 pole designs.

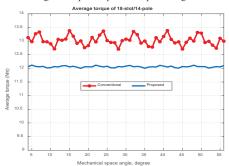


Fig. 10: Output torque for 14 pole designs

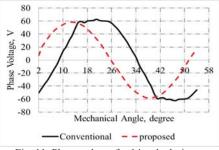


Fig. 11: Phase voltage for 14-pole designs.

to 5.5%). Moreover, core loss and PM eddy loss are strongly related to the stator MMF harmonics, and therefore, the overall core loss reduction is 21% compared to the conventional DLCW. The phase voltage is shown in Fig. 11 where the proposed winding makes the voltages more sinusoidal for the same rotor design. The power factor improves to 0.97 compared to 0.93 for the conventional design. However, the overall average torque reduction in getting the cleaner MMF spectrum is 7%.

IV. CONCLUSION

In this work, a universal winding design method is presented to cancel simultaneously the sub and super harmonics of fractional slot concentrated winding AC machines. The method is based on three single-layer concentrated windings which are divided into two winding sets shifted in space. Application of the proposed method to widely used 12-slot/10-pole machines shows an advantage of 88% torque ripple reduction, 23% THD reduction, and 40% loss

reduction at the cost of 3% loss of average torque due to 5% reduction of torque producing MMF. The winding method has also been applied to the 18-slot/14-pole machine to demonstrate the universality of the concept. FEA results shows a torque ripple improvement of 90%, core loss improvement of 21% with 80% reduction in THD. Future work from this research will include experimental results.

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