A Passivity-Based Globally Stabilizing PI Controller for Primary Control of Radial Power Distribution Systems

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I. Goals

✔ A design of a PI controller for SST-driven distribution microgrids in a radial distribution network such that, for any choice of load, each distribution grid can *globally*:

- Regulate its Low and High DC bus voltages.
- Perform Volt-VAR and volt-Watt Power control.

✔ The proposed controller admits different communication topologies when implemented: fully decentralized, sparsely distributed and all-to-all connected topologies.

✔ The results are validated using a networked microgrid model.
II. System & Model

Power Distribution System

[Diagram of power distribution system with labels for Grid, LVDC bus, AC bus, ac-dc converter, dc-dc converter, SST, and mu grid components.]
ith SST-Driven microgrid

Consideratio
ns

1. The DESD is modelled as a controlled-current source.
2. The DAB converter is modelled as a gyrator [1].
3. Four control inputs
   - $u_d, u_q$: inverter’s duty cycles.
   - $u_g$: DAB converter’s phase shift.
   - $u_b$: battery current.

ith-microgrid model equations:

\[
\begin{align*}
L_{1,i} \frac{di_{1d,i}}{dt} &= -r_{1,i}i_{1d,i} + L_{1,i} \omega i_{1q,i} + v_{c1d,i} - v_{d,i} \\
L_{1,i} \frac{di_{1q,i}}{dt} &= -L_{1,i} \omega i_{1d,i} - r_{1,i}i_{1q,i} + v_{c1q,i} - v_{q,i} \\
C_{1,i} \frac{dv_{c1d,i}}{dt} &= -i_{1d,i} + i_{2d,i} + C_{1,i} \omega v_{c1q,i} \\
C_{1,i} \frac{dv_{c1q,i}}{dt} &= -i_{1q,i} + i_{2q,i} - C_{1,i} \omega v_{c1d,i} \\
L_{2,i} \frac{di_{2d,i}}{dt} &= -r_{2,i}i_{2d,i} + L_{2,i} \omega i_{2q,i} - v_{c1d,i} + u_{d,i} v_{c2,i} \\
L_{2,i} \frac{di_{2q,i}}{dt} &= -L_{2,i} \omega i_{2d,i} - r_{2,i}i_{2q,i} - v_{c1q,i} + u_{q,i} v_{c2,i} \\
C_{2,i} \frac{dv_{c2,i}}{dt} &= -\frac{1}{2} u_{d,i}i_{2d,i} - \frac{1}{2} u_{q,i}i_{2q,i} - u_{g,i} v_{c3,i} \\
C_{3,i} \frac{dv_{c3,i}}{dt} &= -\frac{v_{c3,i}}{r_{L,i}} + u_{g,i} v_{c2,i} + u_{b,i} \\
\end{align*}
\]

Line voltajes are:

\[
\begin{align*}
v_{d,i} &= v_{dg} + \sum_{k=1}^{i} r_{1,k} \sum_{j=k}^{i} i_{1d,j} - \sum_{k=1}^{i} \omega L_{1,k} \sum_{j=k}^{i} \omega i_{1q,j} \\
v_{q,i} &= v_{qg} + \sum_{k=1}^{i} r_{1,k} \sum_{j=k}^{i} i_{1q,j} + \sum_{k=1}^{i} \omega L_{1,k} \sum_{j=k}^{i} \omega i_{1d,j} \\
\end{align*}
\]
The n-microgrid distribution system:

\[ \mathcal{D} \dot{x} = (J - R)x + \mathcal{G}(x)u - [B_d \ B_q] \nu^{dq} \]

where:

\[ x = \text{col}(x_1, \ldots, x_n) \]
\[ u = \text{col}(u_1, \ldots, u_n) \]
\[ x_i = \text{col}(i_{1d,i} i_{1q,i} v_{c1d,i} v_{c1q,i} i_{2d,i} i_{2q,i} v_{c2,i} v_{c3,i}) \]
\[ u_i = \text{col}(u_{d,i} u_{q,i} u_{g,i} 2u_{b,i}) \]
\[ D = \text{diag}(D_1, \ldots, D_n) \]
\[ D_i = \text{diag}(L_{1,i}, L_{1,i}, C_{1,i}, C_{1,i}, L_{2,i}, L_{2,i}, C_{2,i}, C_{3,i}) \]
\[ \mathcal{G} = \text{diag}(G_1, \ldots, G_n) \]
\[ G_i = [J_1 x_i \ | \ J_2 x_i \ | \ J_3 x_i \ | \ e_8] \]
\[ J = \text{diag}(J_{o,1}, \ldots, J_{o,n}) \]

\[ J_{o,i} = -J_{o,i}^T = \begin{bmatrix} L_{1,i} & 0 & 0 & 0 \\ -I_2 & C_{1,i} & I_2 & 0 \\ 0 & -I_2 & L_{2,i} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ J_1 = e_5 e_7^T - e_7 e_5^T, \quad J_2 = e_6 e_7^T - e_7 e_6^T, \quad J_3 = 2(e_8 e_7^T - e_7 e_8^T) \]

\[ B_d = [e_1 \otimes e_1 \ | \ \cdots \ | \ e_n \otimes e_1] \]
\[ B_q = [e_1 \otimes e_2 \ | \ \cdots \ | \ e_n \otimes e_2] \]
\[ \nu^{dq} = \begin{bmatrix} R_{\Sigma} & \omega L_{\Sigma} \\ -\omega L_{\Sigma} & R_{\Sigma} \end{bmatrix} \begin{bmatrix} B_d^T \nabla x \\ B_q^T \nabla x \end{bmatrix} + \begin{bmatrix} 1_n \nu_{g,d} \\ 1_n \nu_{g,q} \end{bmatrix} \]
III. System Control

Control Objectives

1. Regulate LVDC and HVDC bus voltages $v_{c2,i}$ and $v_{c3,i}$ in each microgrid.

2. Perform volt–VAR and volt–Watt control by regulating currents $i_{1d,i}$ and $i_{1q,i}$. 

![Diagram of electrical system control](image-url)
“Standard” PI Control*

HVDC, $i_{1d,i}$ and $i_{1q,i}$ control

LVDC control

The plot contains two closed-loop system eigenvalues for two PI gains sets (red and blue).

For the red set the eigenvalues stay in the LHP for the load variation. However, they approach to the RHP.

For the blue set the eigenvalues migrate to the RHP as load increases: necessity of retuning the gains!
The Proposed PI controller

- It does not need to retune gains for load variations.
- Conditions on the PI gains guaranteeing *global* stability in the model are given.
- It admits different communication topologies.
- Its derivation relies on *passivity theory* and *Lyapunov’s stability theorems*. 
The aforementioned n-microgrid distribution system with output:

\[ y = \mathcal{G}^\top (x_*) x \]

Satisfies the inequality

\[ \dot{V} \leq \tilde{y}^\top \tilde{u} \]

where \( \tilde{\cdot} := (\cdot) - (\cdot)_* \) are the steady-state closed-loop values for the state and control input. Also, \( \tilde{y} \) is the storage function

\[ V = \frac{1}{2} \tilde{x}^\top D \tilde{x} \]

and

\[ \tilde{y} = \text{col} \{ \phi_i(x_i) \} \]

\[ \phi_i(x_i) := \begin{bmatrix} v_{c2,i}^* i_{d2,i} - i_{d2,i}^* v_{c2,i} \\ v_{c2,i}^* i_{q2,i} - i_{q2,i}^* v_{c2,i} \\ 2(v_{c2,i}^* v_{c3,i} - v_{c3,i}^* v_{c2,i}) \\ v_{c3,i} - v_{c3,i}^* \end{bmatrix} \]
What does it mean?...

- Function $V$ is the incremental energy (stored by the system capacitors and inductors) with respect to the steady-state.

- Its time derivative $\dot{V}$ is the rate at with the incremental energy increases.

- The product $\tilde{y}^T \tilde{u}$ is incremental power flowing into the system from sources (DESD and grid voltage).

- Thus, the inequality means that the system’s incremental energy is not greater than the provided one from the sources.
PI controller

Consider the $n$-microgrid distribution system in closed-loop with PI the controller

$$u = -K_p\bar{y} - K_I \int_0^t \bar{y}(\tau)d\tau$$

where

$$\bar{y} := Q_1 K_I \hat{y}$$

and symmetric matrices $K_P, K_I, Q_I > 0$ are such that

$$\frac{1}{2}[K_P Q_I K_I + (K_P Q_I K_I)^	op] > 0$$

Then, the closed-loop trajectories of the closed-loop system are bounded and $x \to x_*$.

Proof sketch: We consider the Lyapunov Function candidate:

$$V_{cl} = V + \frac{1}{2} \hat{z}^\top Q_I^{-1} \hat{z}$$

where $\hat{z} = \bar{y}$. From the time derivative and LaSalle Invariance principle, the claim is proved.
Communication Topologies

- The proposed PI controller

\[ u = -K_p \bar{y} - K_I \int_0^t \bar{y}(\tau) d\tau \]

with \( \bar{y} := Q_I K_I \hat{y} \), admits different communication topologies depending on how \( K_I, Q_I \) and \( K_p \) are defined.

- Selection of these matrices established the dependence of each local controller \( u_i \) on non-local variables.
Three Different Topologies

a. Centralized

b. Descentralized

c. Sparse
Particularly, defining

\[
Q_I = \text{diag}(Q_{I,1}, \ldots, Q_{I,n})
\]

\[
K_P = \text{diag}(K_{P,1}, \ldots, K_{P,n})
\]

\[
K_I = \text{diag}(K_{I,1}, \ldots, K_{I,n})
\]

\[
Q_{I,i} = K_{I,i}^{-1}
\]

leads to the decentralized controller, each local controller has the form:

\[
u_i = -K_{P,i} \phi_i - K_{I,i} \int_0^t \phi_i(x_i(\tau)) d\tau
\]

where \(K_{P,i}, K_{I,i}\) are 4-by-4 diagonal matrices.
- Implementation
Simulation Results: Three SST-driven microgrid system

<table>
<thead>
<tr>
<th>Element</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input grid</td>
<td></td>
<td>3.6 kV, 60 Hz</td>
</tr>
<tr>
<td>Tie-line Impedances</td>
<td></td>
<td>$r_{l,1} = 0.7 \Omega$, $L_{l,1} = 0.7 \text{mH}$</td>
</tr>
<tr>
<td></td>
<td>$r_{l,2} = 1 \Omega$, $L_{l,2} = 1 \text{mH}$</td>
<td></td>
</tr>
<tr>
<td>$Z_{l,3}$</td>
<td>$r_{l,3} = 0.6 \Omega$, $L_{l,3} = 0.7 \text{mH}$</td>
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</tr>
<tr>
<td>Load</td>
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<td></td>
</tr>
<tr>
<td>SST$_1$</td>
<td></td>
<td>40 kW</td>
</tr>
<tr>
<td>SST$_2$</td>
<td></td>
<td>20 kW ($t &lt; 2.5$ s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 kW ($t &gt; 2.5$ s)</td>
</tr>
<tr>
<td>SST$_3$</td>
<td></td>
<td>16 kW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SST #</th>
<th>Current references</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SST$_1$</td>
<td>$i_{2d,1}^* = -19.31 \text{A}$, $i_{2q,1}^* = 0.06 \text{A}$</td>
<td></td>
</tr>
<tr>
<td>SST$_2$</td>
<td>$i_{2d,2}^* = -8.61 \text{A}$, $i_{2q,2}^* = 0.07 \text{A}$</td>
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<tr>
<td>SST$_3$</td>
<td>$i_{2d,3}^* = -6.77 \text{A}$, $i_{2q,3}^* = 0.07 \text{A}$</td>
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</tr>
</tbody>
</table>

For the three SST microgrids the DC buses set-points are: $v_{c2,i}^* = 6.1kV$, $v_{c3,i}^* = 400V$
Simulation Results: Three SST-driven microgrid system

1. Load in SST2 increases 200% at $t = 2.5s$
2. The d-axis reference of SST3 changes to $i_{2d,3*} = 3A$
3. PI gains remain unchanged at any time.

PQ regulation
In this simulation, centralized topology makes the inverter control input signal of SST2 stay within the non-saturated region.
“Conventional” PI vs Proposed PI in a *single* SST-system
THANK YOU