FREEDM SYSTEMS CENTER

A Passivity-Based Globally Stabilizing PI Controller for Primary Control of Radial Power Distribution Systems

Alireza A. Milani
Rafael Cisneros
Aranya Chakraborty
Iqbal Husain

I. Goals

- ✔ A design of a PI controller for SST-driven distribution microgrids in a radial distribution network such that, for any choice of load, each distribution grid can *globally*:

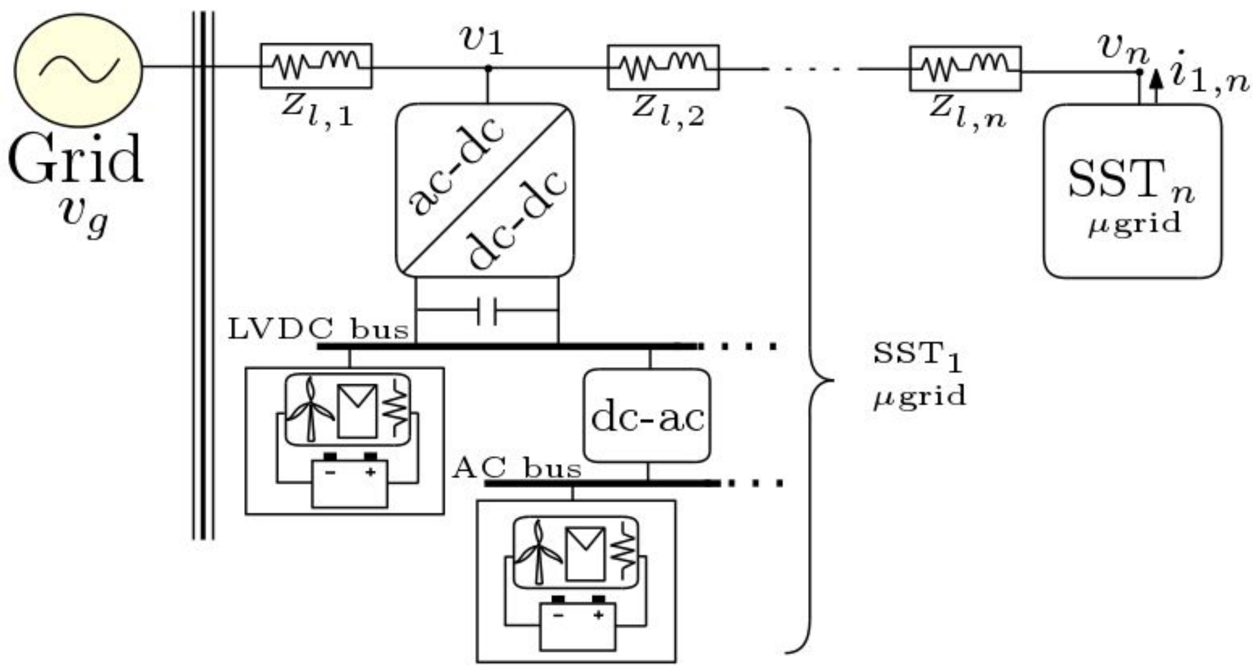
    - Regulate its Low and High DC bus voltages.
    - Perform Volt-VAR and volt-Watt Power control.

- ✔ The proposed controller admits different communication topologies when implemented: fully decentralized, sparsely distributed and all–to–all connected topologies.

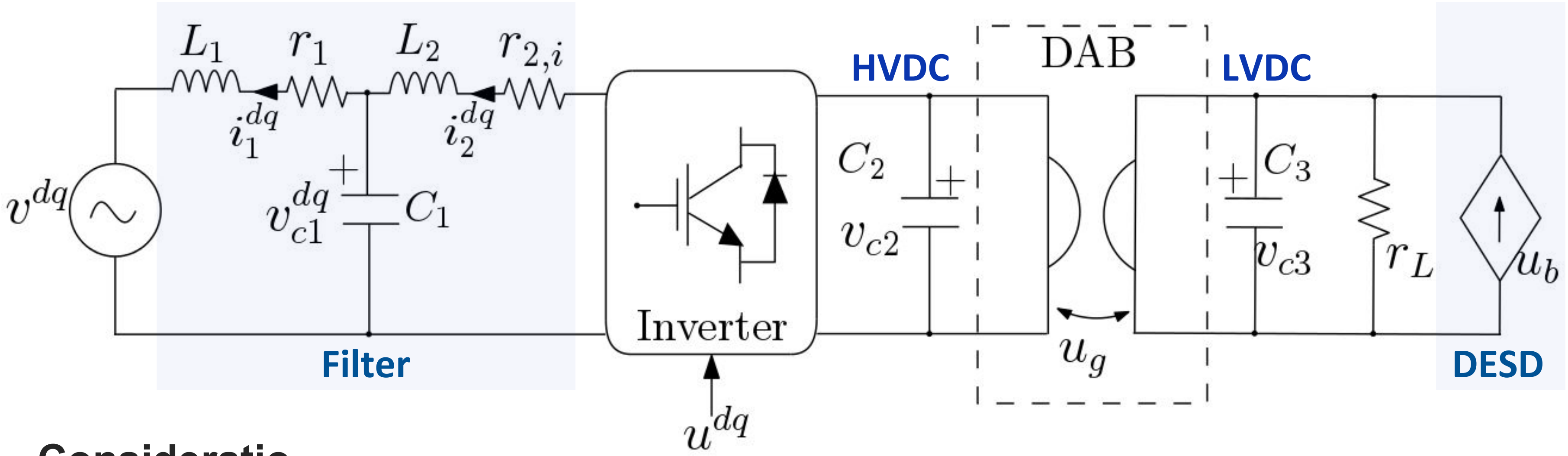
- ✔ The results are validated using a networked microgrid model.

II. System & Model

Power Distribution System



ith SST-Driven microgrid



Considerations

1. The DESD is modelled as a controlled−current source.
2. The DAB converter is modelled as a gyrator [1].
3. Four control inputs
   - u_d, u_q: inverter's duty cycles.
   - u_g : DAB converter's phase shift.
   - u_b : battery current.

[1] E. Ehsani, I. Husain and M. O. Bilgic, *Power converters as natural gyrators*, IEEE Trans. On Circuits Syst. I Fundam. Theory Appl., vol. 40, no. 12, pp. 946-949, 1993

ith-microgrid model equations:

$$
\begin{aligned}
L_{1,i}\dot{i}_{1d,i} &= -\, r_{1,i} i_{1d,i} + L_{1,i}\omega\, i_{1q,i} + v_{c1d,i} - \, v_{d,i} \\
L_{1,i}\dot{i}_{1q,i} &= -\, L_{1,i}\omega\, i_{1d,i} - r_{1,i} i_{1q,i} + v_{c1q,i} - \, v_{q,i} \\
C_{1,i}\dot{v}_{c1d,i} &= -\, i_{1d,i} + i_{2d,i} + C_{1,i}\omega v_{c1q,i} \\
C_{1,i}\dot{v}_{c1q,i} &= -\, i_{1q,i} + i_{2q,i} - C_{1,i}\omega v_{c1d,i} \\
L_{2,i}\dot{i}_{2d,i} &= -\, r_{2,i} i_{2d,i} + L_{2,i}\omega i_{2q,i} - v_{c1d,i} + u_{d,i}\, v_{c2,i} \\
L_{2,i}\dot{i}_{2q,i} &= -\, L_{2,i}\omega i_{2d,i} - r_{2i} i_{2q,i} - v_{c1q,i} + u_{q,i}\, v_{c2,i} \\
C_{2,i}\dot{v}_{c2,i} &= -\, \frac{1}{2} u_{d,i} i_{2d,i} - \frac{1}{2} u_{q,i} i_{2q,i} - u_{g,i} v_{c3,i} \\
C_{3,i}\dot{v}_{c3,i} &= -\, \frac{v_{c3,i}}{r_{L,i}} + u_{g,i} v_{c2,i} + u_{b,i}.
\end{aligned}
$$

Line voltajes are:

$$
\left\{
\begin{aligned}
v_{d,i} &= v_{dg} + \sum_{k=1}^{i} r_{l,k} \sum_{j \geq k} i_{1d,j} - \sum_{k=1}^{i} \omega L_{l,k} \sum_{j \geq k} i_{1q,j} \\
v_{q,i} &= v_{qg} + \sum_{k=1}^{i} r_{l,k} \sum_{j \geq k} i_{1q,j} + \sum_{k=1}^{i} \omega L_{l,k} \sum_{j \geq k} i_{1d,j}.
\end{aligned}
\right.
$$

FREEDM SYSTEMS CENTER

NSF

The n-microgrid distribution system:

$$\mathcal{D}\dot{x} = (\mathcal{J} - \mathcal{R})x + \mathcal{G}(x)u - [\mathcal{B}_d \quad \mathcal{B}_q]v^{dq}$$

where
:

$$\begin{aligned}
x &= \mathrm{col}(x_1, \cdots, x_n) \\
u &= \mathrm{col}(u_1, \cdots, u_n) \\
x_i &= \mathrm{col}(i_{1d,i}\ i_{1q,i}\ v_{c1d,i}\ v_{c1q,i}\ i_{2d,i}\ i_{2q,i}\ v_{c2,i}\ v_{c3,i}) \\
u_i &= \mathrm{col}(u_{d,i}\ u_{q,i}\ u_{g,i}\ 2u_{b,i}) \\
\mathcal{D} &= \mathrm{diag}(D_1, \cdots, D_n) \\
D_i &= \mathrm{diag}(L_{1,i}, L_{1,i}, C_{1,i}, C_{1,i}, L_{2,i}, L_{2,i}, C_{2,i}, C_{3,i}) \\
\mathcal{G} &= \mathrm{diag}(G_1, \cdots, G_n) \\
G_i &= [J_1 x_i \mid J_2 x_i \mid J_3 x_i \mid e_8] \\
\mathcal{J} &= \mathrm{diag}(J_{o,1}, \cdots, J_{o,n})
\end{aligned}$$

$$J_{o,i} = -J_{o,i}^\top = \begin{bmatrix} L_{1,i}\omega\mathbb{J} & \mathbb{I}_2 & \mathbb{O}_2 & \mathbb{O}_2 \\ -\mathbb{I}_2 & C_{1,i}\omega\mathbb{J} & \mathbb{I}_2 & \mathbb{O}_2 \\ \mathbb{O}_2 & -\mathbb{I}_2 & L_{2,i}\omega\mathbb{J} & \mathbb{O}_2 \\ \mathbb{O}_2 & \mathbb{O}_2 & \mathbb{O}_2 & \mathbb{O}_2 \end{bmatrix}$$

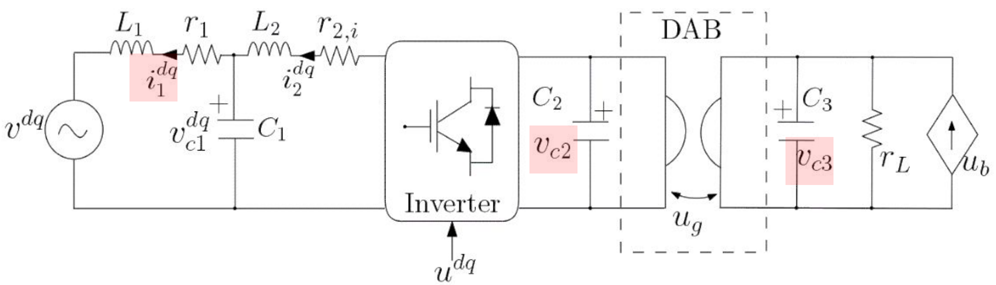
$$\begin{aligned}
J_1 &= e_5 e_7^\top - e_7 e_5^\top, \quad J_2 = e_6 e_7^\top - e_7 e_6^\top, \quad J_3 = 2(e_8 e_7^\top - e_7 e_8^\top) \\
\mathcal{B}_d &= [\mathbf{e}_1 \otimes e_1 \mid \cdots \mid \mathbf{e}_n \otimes e_1] \\
\mathcal{B}_q &= [\mathbf{e}_1 \otimes e_2 \mid \cdots \mid \mathbf{e}_n \otimes e_2] \\
v^{dq} &= \begin{bmatrix} R_\Sigma & \omega L_\Sigma \\ -\omega L_\Sigma & R_\Sigma \end{bmatrix} \begin{bmatrix} \mathcal{B}_d^\top x \\ \mathcal{B}_q^\top x \end{bmatrix} + \begin{bmatrix} \mathbb{1}_n v_{g,d} \\ \mathbb{1}_n v_{g,q} \end{bmatrix}
\end{aligned}$$

III. System Control

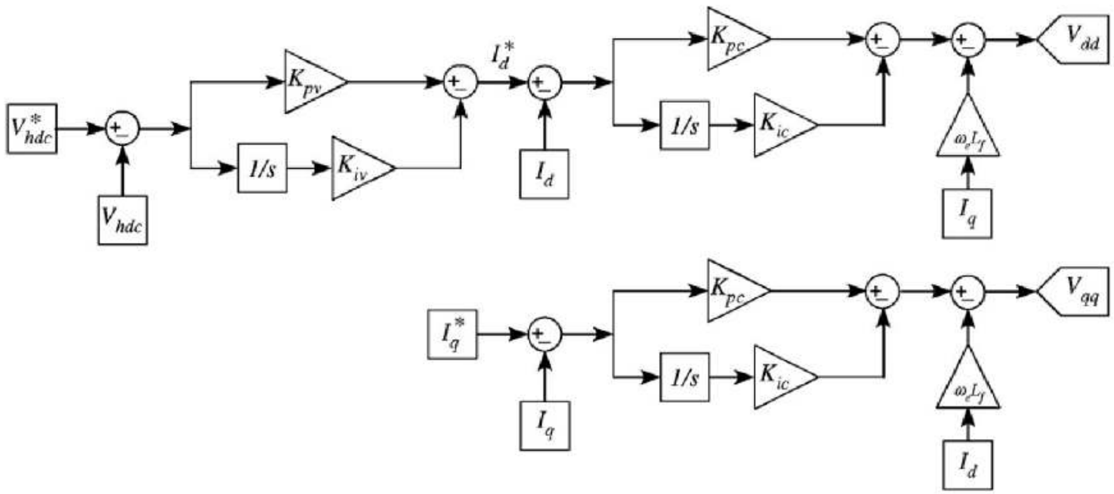
Control Objectives

1. Regulate LVDC and HVDC bus voltajes $v_{c2,i}$ and $v_{c3,i}$ in each microgrid.

2. Perform volt–VAR and volt–Watt control by regulating currents $i_{1d,i}$ and $i_{1q,i}$.

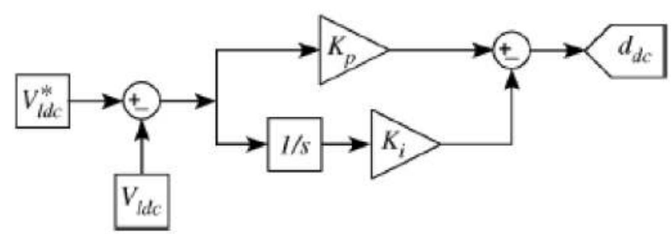


"Standard" PI Control*



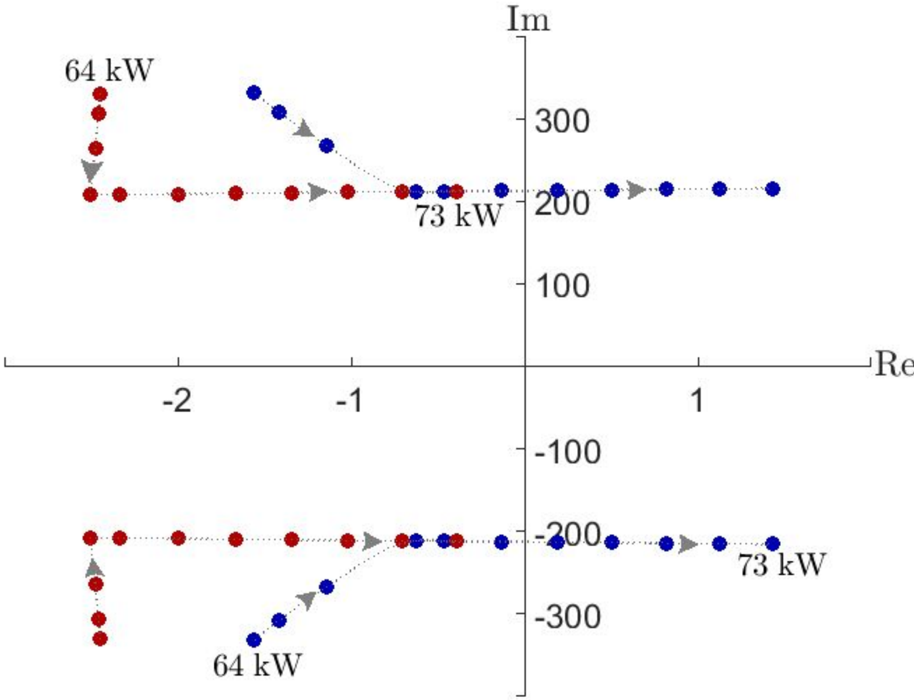
HVDC, $i_{1d,i}$ and $i_{1q,i}$ control

LVDC control



*[2] D. G. Shah and M. L. Crow, *Stability Assessment Extensions for single-Phase Distribution Solid-State Transformers*, IEEE Trans. On Power Delivery., vol. 30, no. 3, 2015

- The plot contains two closed-loop system eigenvalues for two PI gains sets (red and blue).
- For the red set the eigenvalues stay in the LHP for the load variation. However, they approach to the RHP.
- For the blue set the eigenvalues migrate to the RHP as load increases: necessity of retuning the gains!



The Proposed PI controller

- It does not need to retune gains for load variations.
- Conditions on the PI gains guaranteeing *global* stability in the model are given.
- It admits different communication topologies.
- Its derivation relies on **passivity theory** and **Lyapunov's stability theorems**.

FREEDM SYSTEMS CENTER

NSF

Incremental Passivity

The aforementioned n-microgrid distribution system with output:

$$y = \mathcal{G}^\top(x_\star)x$$

Satisfies the inequality

$$\dot{V} \le \tilde{y}^\top \tilde{u}$$

where $\tilde{(\cdot)} := (\cdot) - (\cdot)_\star$ $x_\star, u_\star$ **are the steady-state closed-loop values for the state and control input. Also,** V **is the storage function**

$$V = \frac{1}{2}\tilde{x}^\top \mathcal{D}\tilde{x}$$

and

$$\tilde{y} = \mathrm{col}\{\phi_i(x_i)\} \quad \phi_i(x_i) := \begin{bmatrix} v_{c2,i}^\star i_{2d,i} - i_{2d,i}^\star v_{c2,i} \\ v_{c2,i}^\star i_{2q,i} - i_{2q,i}^\star v_{c2,i} \\ 2(v_{c2,i}^\star v_{c3,i} - v_{c3,i}^\star v_{c2,i}) \\ v_{c3,i} - v_{c3,i}^\star \end{bmatrix}.$$

What does it mean?...

- Function V is the incremental energy (stored by the system capacitors and inductors) with respect to the steady-state.
- Its time derivative $\dot{V}$ is the rate at with the incremental energy increases.
- The product $\tilde{y}^T\tilde{u}$ is incremental power flowing into the system from sources (DESD and grid voltage).
- Thus, the inequality means that the system's incremental energy is not greater tan the provided one from the sources.

PI controller

Consider the n-microgrid distribution system in closed-loop with PI the controller

$$\boxed{u = -K_p\bar{y} - K_I\int_0^t \bar{y}(\tau)d\tau}$$

where

$$\bar{y} := Q_I K_I \tilde{y}$$

and symmetric matrices $K_P, K_I, Q_I > 0$ are such that

$$\frac{1}{2}[K_p Q_I K_I + (K_p Q_I K_I)^\top] > 0$$

Then, the closed-loop trajectories of the closed-loop system are bounded and $x \to x_*$

<u>Proof sketch</u>: We consider the Lyapunov Function candidate:

$$V_{cl} = V + \frac{1}{2}\tilde{z}^\top Q_I^{-1}\tilde{z}$$

where $\dot{\tilde{z}} = \bar{y}$. From the time derivative and LaSalle Invariance principle, the claim is proved.

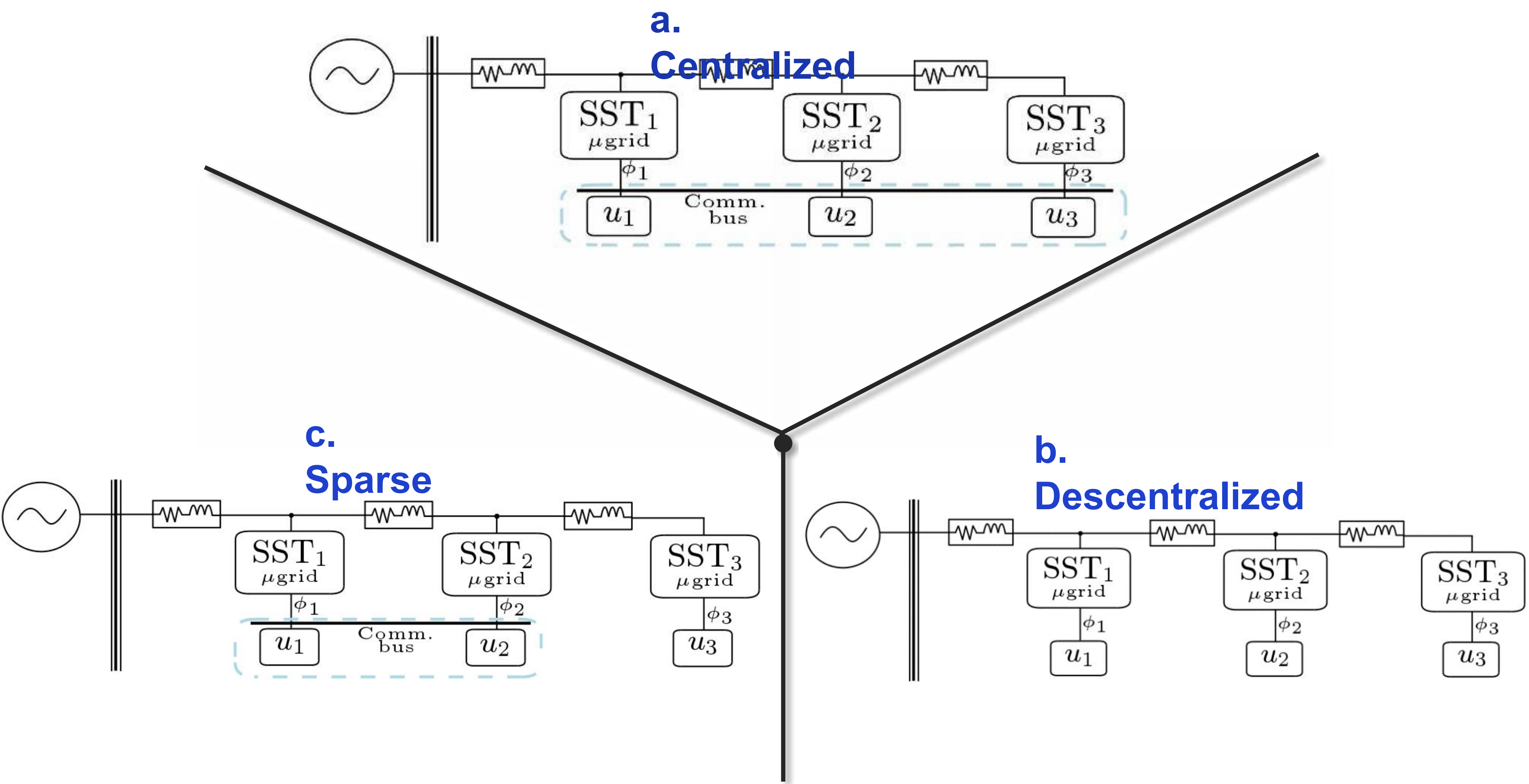
Communication Topologies

- The proposed PI controller

$$\boxed{u = -K_p\bar{y} - K_I \int_0^t \bar{y}(\tau)d\tau}$$

with $\bar{y} := Q_I K_I \tilde{y}$, admits different communication topologies depending on how K_I, Q_I and K_p are defined.

- Selection of these matrices established the dependence of each local controller u_i on non–local variables.



Three Different Topologies

Particularly, defining

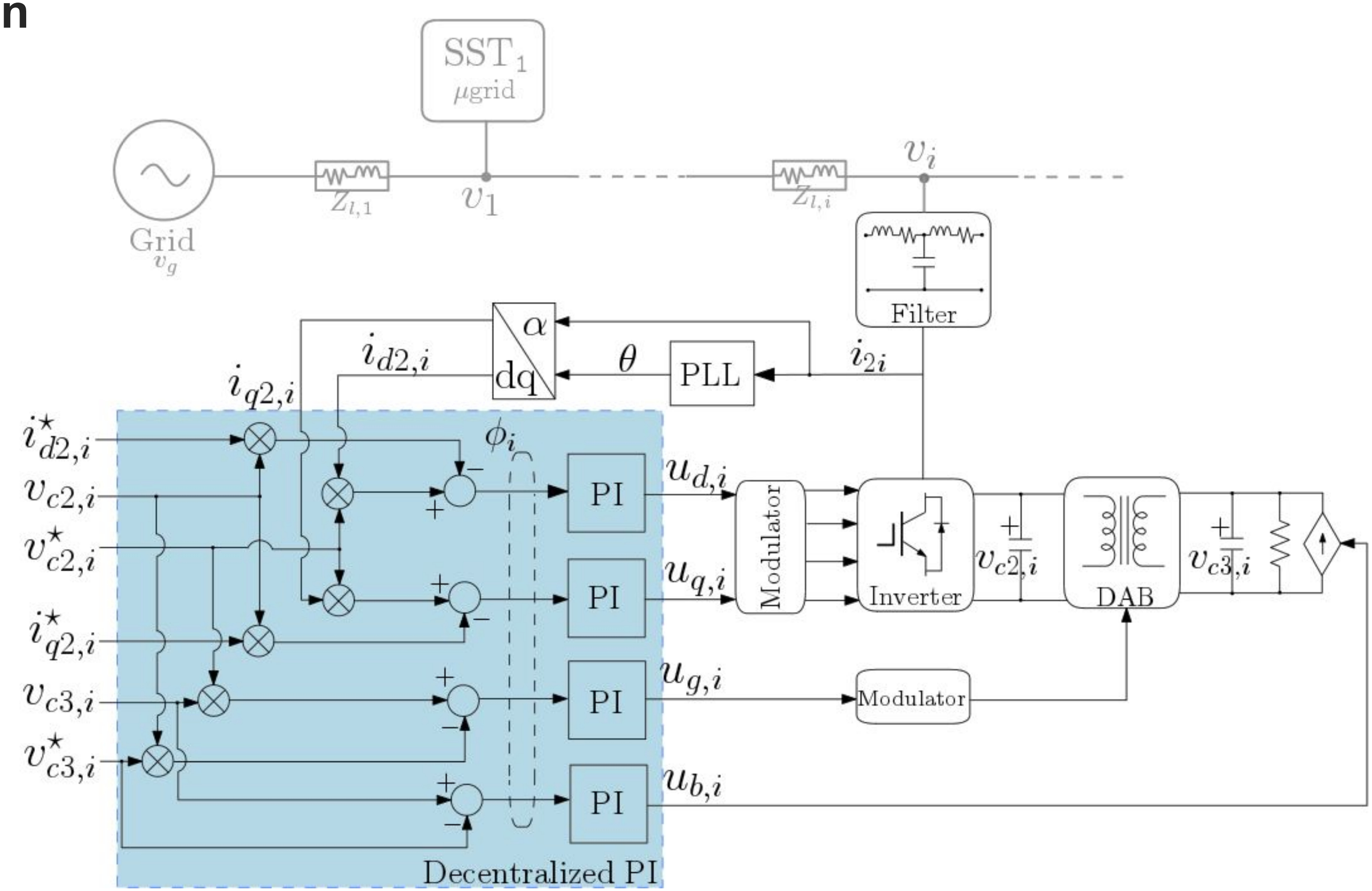
$$\begin{aligned} Q_I &= \mathrm{diag}(Q_{I,i}. \cdots, Q_{I,n}) \\ K_P &= \mathrm{diag}(K_{P,1}, \cdots, K_{P,n}) \\ K_I &= \mathrm{diag}(K_{I,1}, \cdots, K_{I,n}) \\ Q_{I,i} &= K_{I,i}^{-1} \end{aligned}$$

leads to the descentralized controller, each local controller has the form:

$$u_i = -K_{p,i}\phi_i - K_{I,i}\int_0^t \phi_i(x_i(\tau))d\tau$$

where $K_{p,i}$, $K_{I,i}$ are 4–by–4diagonal matrices.

- **Implementation**



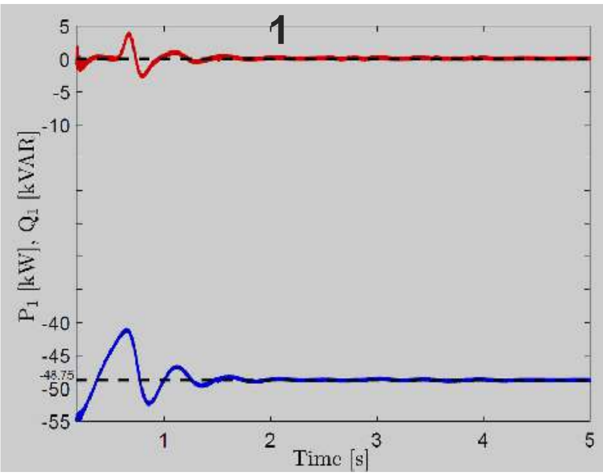
Simulation Results: Three SST-driven microgrid system

Element		Value
Parameters		
Input grid		3.6 kV, 60 Hz
Tie-line Impedances	$Z_{l,1}$	$r_{l,1} = 0.7\ \Omega$, $L_{l,1} = 0.7$ mH
	$Z_{l,2}$	$r_{l,2} = 1\ \Omega$, $L_{l,2} = 1$ mH
	$Z_{l,3}$	$r_{l,3} = 0.6\ \Omega$, $L_{l,3} = 0.7$ mH
Load	SST_1	40 kW
	SST_2	20 kW ($t < 2.5$ s)
		40 kW ($t > 2.5$ s)
	SST_3	16 kW
Current references		
SST #		Value
SST_1		$i^{\star}_{2d,1} = -19.31$ A, $i^{\star}_{2q,1} = 0.06$ A
SST_2		$i^{\star}_{2d,2} = -8.61$ A, $i^{\star}_{2q,2} = 0.07$ A
SST_3		$i^{\star}_{2d,3} = -6.77$ A, $i^{\star}_{2q,3} = 0.07$ A

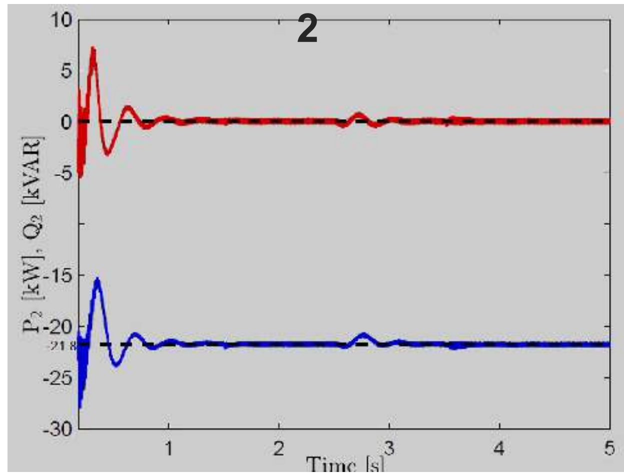
For the three SST microgrids the DC buses set−points are :$v^{*}_{c2,i} = 6.1kV$, $v^{*}_{c3,i} = 400V$

Simulation Results: Three SST-driven microgrid system

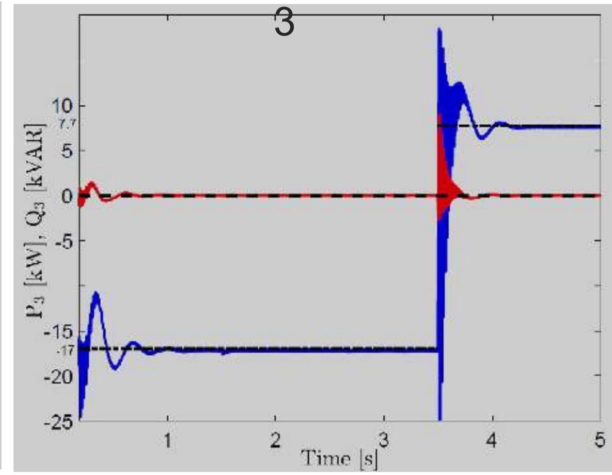
SST 1



SST 2



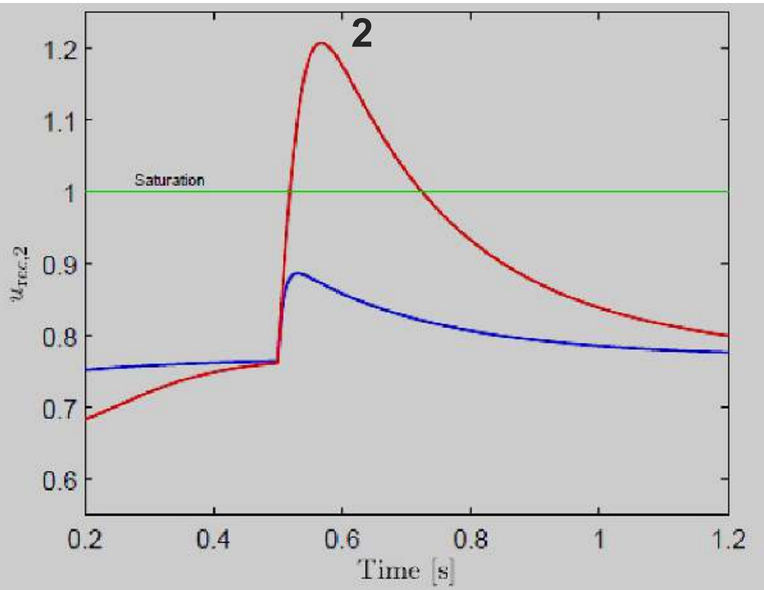
SST 3



1. Load in SST2 increases 200% at $t = 2.5$s
2. The d–axis reference of SST3 changes to $i_{2d,3*} = 3$A
3. PI gains remain unchanged at any time.

PQ regulation

SST
2

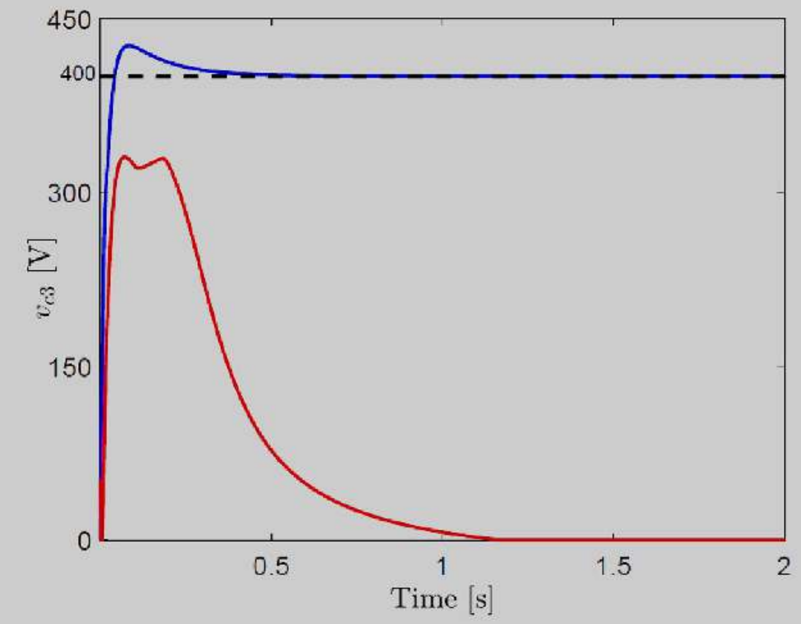


(c) Rectifier duty cycle in SST$_2$: Centralized control, Decentralized control.

✔ In this simulation, centralized topology makes the inverter control input signal of SST2 stay within the non-saturated region.

Centralized vs Descentralized controller

LVDC regulation



(a) Voltage regulation: Proposed PI, Conventional PI. Conventional PI fails to regulate v_{c3}.

“Conventional” PI vs Proposed PI in a *single* SST-system

THANK
YOU